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PHYSICAL ACOUSTICS

Propagation of Detonation Waves in Bubbly Liquids in Suddenly Widening Channels

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Abstract—The propagation of detonation waves in a channel filled with a bubbly liquid that suddenly widens is investigated. Possible scenarios for the dynamics of detonation waves after their transition to the widening part of the channel are analyzed. The influence of the volume content of a combustible gas and the geometric parameters of the channel on the propagation and breakdown of the detonation wave has been established.

Keywords: bubbly liquid, waves, two-dimensional problem, detonation failure, complex channel

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INTRODUCTION

The study of the patterns of pressure wave propagation in bubbly liquids is of great interest among researchers because such systems are widespread in nature and a number of industries use bubble media. The features of wave propagation in a bubbly liquid are related to the cumulative interaction of nonlinear, dispersive, and dissipative effects. In a bubbly liquid, the properties of an almost incompressible liquid (carrier phase) change radically when gas (dispersed phase) is added in a small volume and, moreover, mass (bubbles). A peculiarity of bubble liquids is their high static compressibility while maintaining a high density close to that of the liquid, which in turn leads to a small equilibrial sound speed. An interesting feature of a bubbly liquid in dynamic processes is inertia of the liquid with a change in volume of the mixture due to compression or expansion of bubbles [1, 2]. The anomalously strong compressibility of a bubbly liquid and dissipation of the wave energy leads to interesting effects as waves are reflected and refracted at the boundaries of bubble media [3–6].

A bubble medium with a chemically active gas in the bubbles is an explosive in which detonation waves can occur with an amplitude tens of times greater than that of the initial signal [7–13]. Therefore, the study of detonation waves in bubble media is interesting both for explosion-proofing and for information transmission in a liquid by means of waves. There are also recent papers on detonation waves used in pulsed detonation hydrojet propulsion [14].

Studies [7–17] are devoted to one-dimensional detonation waves, the results of which are summarized in monographs [18–20]; at present, two-dimensional detonation waves in bubbly liquids are being actively studied.

The authors of [21] study the explosion of a finite-sized bubble zone in the bulk of a liquid. It is shown that, due to focusing of the pressure wave near the bubble zone, the amplitude of the initial wave capable of initiating bubble detonation is significantly reduced.

The dynamics of detonation waves along a tubular volume of a bubbly liquid containing a chemically active gas mixture was considered in [22]. It is shown that such a bubble cluster can serve as a waveguide for transmitting pulsed signals—detonation solitons. So that detonation solitons do not break down due to acoustic wave radiation into the liquid surrounding the waveguide, the radius of the waveguide must exceed some critical value depending on the bubble radius, bubble volume content, and characteristics of the explosive gas mixture. The authors of [23] applied the two-phase Jordanian–Kogarko model to formulate and numerically solve the problem of a detonation wave propagating in the cylindrical column of a chemically active bubble medium that shields the liquid from the tube walls. The wave structure of the reaction zone and the detonation rate of the column with the bubble medium were calculated. It was established that a self-sustaining wave can propagate at a velocity 1.5–2.5 times faster than that of a one-dimensional bubble detonation.

In [24], it was found that a detonation wave can propagate through a two-layer bubble mixture whose velocity is less than in a single-layer bubble system. The two-dimensional structure of a two-layer bubble

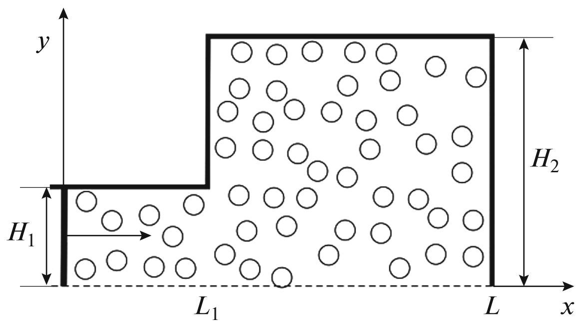

Fig. 1. Flow pattern.

detonation was obtained and analyzed. It is shown that for a channel width smaller than the characteristic wavelength, the velocity of a two-layer wave can be determined from a one-dimensional model of a two-component bubble mixture.

A numerical study of bubble detonation in channels of complex shape was carried out in [25], which examined peculiarities in the dynamics of detonation waves in a bubbly liquid in narrowing and widening channels.

The aim of this study is to numerically simulate propagation of a detonation wave in a bubbly liquid in a channel that suddenly expands.

FOMULATION OF THE PROBLEM AND BASIC EQUATIONS

Let us onsider a flat channel consisting of narrow and wide parts, filled with a homogeneous bubble medium with a combustible gas mixture (e.g., a mixture of acetylene with oxygen or an explosive gas). It is assumed that the channel is symmetric about the x axis; therefore, it suffices to consider its upper half (Fig. 1). A plane detonation wave propagates in the bubbly liquid from left to right along the narrow part of the channel. We study the process of its transition from the narrow to the wide part of the channel and its subsequent propagation along this part. In the flow diagram (Fig. 1), L_1 is the length of the narrow part of the channel, L is the length of the computation domain, H_1 is half the transverse dimension of the narrow part of the channel, and H_2 is half the transverse size of the wide part. Note that Fig. 1 shows a simplified formulation of the problem, since it is technically difficult to create a bubble medium of uniform concentration in a channel with a variable cross section.

To describe wave motion, using general assumptions for bubbly liquids, we write the system of macroscopic equations of mass, number of bubbles, pulses, and pressure in the bubbles [1]:

$$\frac{d\rho_i}{dt} + \rho_i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \quad (i = l, g),$$
$$\frac{dn}{dt} + n\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0, \quad \rho\frac{du}{dt} + \frac{\partial p_l}{\partial x} = 0,$$
$$\rho\frac{dv}{dt} + \frac{\partial p_l}{\partial y} = 0, \quad \rho = \rho_g + \rho_l, \tag{1}$$
$$\left(\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right), \quad \alpha_l + \alpha_g = 1,$$
$$\rho_i = \rho_i^0\alpha_i, \quad \alpha_g = \frac{4}{3}\pi n a^3,$$

where ρ_i^0, α_i, p_l, n, and a are, respectively, the density, volume content of the ith phase, pressure of the carrier liquid, number and radius of bubbles, and u and v are the velocity projections onto the x and y axes, respectively. Subscripts $i = l, g$ denote the parameters of the liquid and gas phases.

When describing radial motion, we assume that $w = w_A + w_R$, where w_R is determined from the Rayleigh–Lamb equation and w_A is determined from the solution to the problem of spherical unloading on a sphere of radius a in the carrier liquid in the acoustic approximation [26]:

$$a\frac{dw_R}{dt} + \frac{3}{2}w_R^2 + 4\nu_l\frac{w_R}{a} = \frac{p_g - p_l}{\rho_l^0},$$
$$w_A = \frac{p_g - p_l}{\rho_l^0 C_l \alpha_g^{1/3}},$$

where ν_l is viscosity of the liquid C_l is the sound speed in a "pure" liquid.

We assume that the liquid is acoustically compressible and the gas is calorically perfect,

$$p_l = p_0 + C_l^2(\rho_l^0 - \rho_{l0}^0), \quad p_g = \rho_g^0 B T_g,$$

where B is the gas constant. Here and below, the subscript 0 refers to the initial unperturbed state.

To describe the intensity of interfacial heat transfer, we use a scheme that takes into account phase slip [12]. When taking into account phase slip, we assume that the surface of a bubble is renewed and heat flow is determined by the thermal conductivity of the liquid:

$$q = \mathrm{Nu}_l\lambda_l\frac{T_g - T_0}{2a}, \quad \frac{T_g}{T_0} = \frac{p_g}{p_0}\left(\frac{a}{a_0}\right)^3,$$
$$\mathrm{Nu}_l = 0.65\sqrt{\mathrm{Pe}_l}, \quad \mathrm{Pe}_l = \frac{2a|v_{lg}|}{k_l}, \quad k_l = \frac{\lambda_l}{\rho_l^0 c_l}. \tag{2}$$

Here, $T_0 = \text{const}$ is the temperature of the liquid, v_{lg} is the relative phase velocity, Nu_i and Pe_i are the Nusselt and Peclet numbers for the phases, and c_l, λ_1, and k_l are the heat capacity, thermal conductivity, and thermal diffusivity coefficient of the liquid.

To determine the relative phases velocity, we can write the following equation [1]:

$$\frac{\partial \mathbf{v}_{lg}}{\partial t} = -2\frac{\partial \mathbf{v}}{\partial t} - \frac{3}{a} w v_{lg} - \frac{3\mathbf{f}}{2\pi a^3 \rho_l^0}, \qquad (3)$$

where $\mathbf{v} = u\mathbf{i} + v\mathbf{j}$, $\mathbf{v}_{lg} = u_{lg}\mathbf{i} + v_{lg}\mathbf{j}$, $\mathbf{i}, \mathbf{j}$ are the unit axes x and y and $\mathbf{f}$ is the viscous friction force.

The viscous friction force is taken as

$$\mathbf{f} = \frac{1}{2} C_D \pi a^3 \mathbf{v}_{lg} \left| v_{lg} \right|.$$

The resistance coefficient C_D is given in the following form [33]:

$$C_D = \begin{cases} \dfrac{48}{\mathrm{Re}}, & 0 \le \mathrm{Re} < 180, \\ \dfrac{\mathrm{Re}^{4/3}}{10^{3.6}}, & \mathrm{Re} > 180, \end{cases} \qquad \mathrm{Re} = \frac{2a\left|v_{lg}\right|}{v_l},$$

where v_l is the kinematic viscosity of the liquid and Re is the Reynolds number.

We assume that the temperature of the gas inside the bubbles upon reaching a certain value T_* instantly changes to ΔT, corresponding to the calorific value of the gas, as a result of which the pressure in the gas increases. Physically, this means that the induction period of chemical reactions is significantly less than the characteristic bubble pulsation time.

A stoichiometric acetylene–oxygen mixture $C_2H_2 + 2.5O_2$ is taken as the gas phase for calculations. This gas phase was chosen because it has been used in most experiments [7–9]. As a liquid phase, it is an aqueous glycerin solution with a glycerin volume content of 0.5 [7–9].

To numerically analyze the problem of propagation of detonation waves in a bubbly liquid in a suddenly expanding channel, it is more convenient to use system of equations (1)–(3) written in Lagrangian coordinates [27]. This is because, in Lagrangian coordinates, the original boundaries of the inhomogeneities remain fixed.

We introduce a system of equations in Lagrangian coordinates:

$$\frac{\partial p_l}{\partial t} = \frac{C_l^2 \rho_l^0}{(1-\alpha_g)}\left[\frac{3\alpha_g}{a} w - \left(\frac{\alpha_g}{J} + \frac{\rho_{l0}}{J^2 \rho_l^0}\right)\frac{\partial J}{\partial t}\right],$$

$$\frac{\partial \alpha_g}{\partial t} = \frac{3\alpha_g}{a} w - \frac{\alpha_g}{J}\frac{\partial J}{\partial t}, \quad \frac{\partial u}{\partial t} = -\frac{1}{J\rho}\left(\frac{\partial p_l}{\partial x_0}\frac{\partial y}{\partial y_0} - \frac{\partial p_l}{\partial y_0}\frac{\partial y}{\partial x_0}\right),$$

$$\frac{\partial x}{\partial t} = u, \quad \frac{\partial v}{\partial t} = -\frac{1}{J\rho}\left(\frac{\partial p_l}{\partial y_0}\frac{\partial x}{\partial x_0} - \frac{\partial p_l}{\partial x_0}\frac{\partial x}{\partial y_0}\right), \quad \frac{\partial y}{\partial t} = v,$$

$$\frac{\partial p_g}{\partial t} = -\frac{3\gamma p_g}{a} w - \frac{3(\gamma - 1)}{a_0} q, \quad \frac{\partial a}{\partial t} = w = w_R + w_A,$$

$$\frac{\partial w_R}{\partial t} = \frac{1}{a}\left[\frac{p_g - p_l}{\rho_l^0} - \frac{3}{2} w_R^2 - 4\nu_l \frac{w_R}{a}\right],$$

$$w_A = \frac{p_g - p_l}{\rho_l^0 C_l \alpha_g^{1/3}}, \quad \left|\nu_{lg}\right| = \sqrt{u_{lg}^2 + v_{lg}^2},$$

$$\frac{\partial u_{lg}}{\partial t} = -2\frac{\partial u}{\partial t} - \frac{3}{a} w u_{lg} - \frac{3 f_x}{2\pi a^3 \rho_l^0},$$

$$\frac{\partial v_{lg}}{\partial t} = -2\frac{\partial v}{\partial t} - \frac{3}{a} w v_{lg} - \frac{3 f_y}{2\pi a^3 \rho_l^0}, \qquad (4)$$

$$f_x = \frac{1}{2} C_D \pi a^3 u_{\mathrm{lg}} \left|\nu_{\mathrm{lg}}\right|, \quad f_y = \frac{1}{2} C_D \pi a^3 v_{\mathrm{lg}} \left|\nu_{\mathrm{lg}}\right|,$$

$$C_D = \begin{cases} \dfrac{48}{\mathrm{Re}}, & 0 \le \mathrm{Re} < 180, \\ \dfrac{\mathrm{Re}^{4/3}}{10^{3.6}}, & \mathrm{Re} > 180, \end{cases} \qquad \mathrm{Re} = \frac{2a\left|\nu_{lg}\right|}{\nu_l},$$

$$q = \mathrm{Nu}_l \lambda_l \frac{T_g - T_0}{2a}, \quad \frac{T_g}{T_0} = \frac{p_g}{p_0}\left(\frac{a}{a_0}\right)^3,$$

$$\mathrm{Nu}_l = 0.65\sqrt{\mathrm{Pe}_l}, \quad \mathrm{Pe}_l = \frac{2a\left|\nu_{\mathrm{lg}}\right|}{k_l}, \quad k_l = \frac{\lambda_l}{\rho_l^0 c_l}$$

$$J = \frac{\partial x}{\partial x_0}\frac{\partial y}{\partial y_0} - \frac{\partial x}{\partial y_0}\frac{\partial y}{\partial x_0}, \quad \frac{\partial J}{\partial t} = \frac{\partial u}{\partial x_0}\frac{\partial y}{\partial y_0} - \frac{\partial u}{\partial y_0}\frac{\partial y}{\partial x_0} + \frac{\partial x}{\partial x_0}\frac{\partial v}{\partial y_0} - \frac{\partial x}{\partial y_0}\frac{\partial v}{\partial x_0}.$$

System of equations (4) was solved numerically with an explicit scheme [27]. Taking into account the interfacial heat transfer and acoustic unloading of bubbles, the equations are a system with fairly strong natural dissipation; therefore, no artificial viscosity is required.

INITIAL AND BOUNDARY CONDITIONS

The conditions for $t = 0$, corresponding to the initial state of a homogeneous bubble mixture in the channel is written as

$$u = v = 0, \quad p_l = p_0, \quad p_g = p_0, \quad a = a_0,$$
$$w = 0, \quad T_g = T_0, \quad \alpha_g = \alpha_{g0}, \quad \rho = \rho_{l0}^0 (1 - \alpha_{g0}).$$

The initiating pulse at the boundary of the bubbly liquid ($x_0 = 0$) is given in the form of a bell-shaped time law for the speed of a hard striker. The corresponding boundary condition is written as

$$u(t, r_0) = \begin{cases} \Delta u_0 \exp\left(-\left(\dfrac{t - t_*/2}{t_*/6}\right)^2\right), & 0 < t < t_*, \\ 0, \quad t > t_* \end{cases},$$
$$\text{for} \quad x_0 = 0,$$

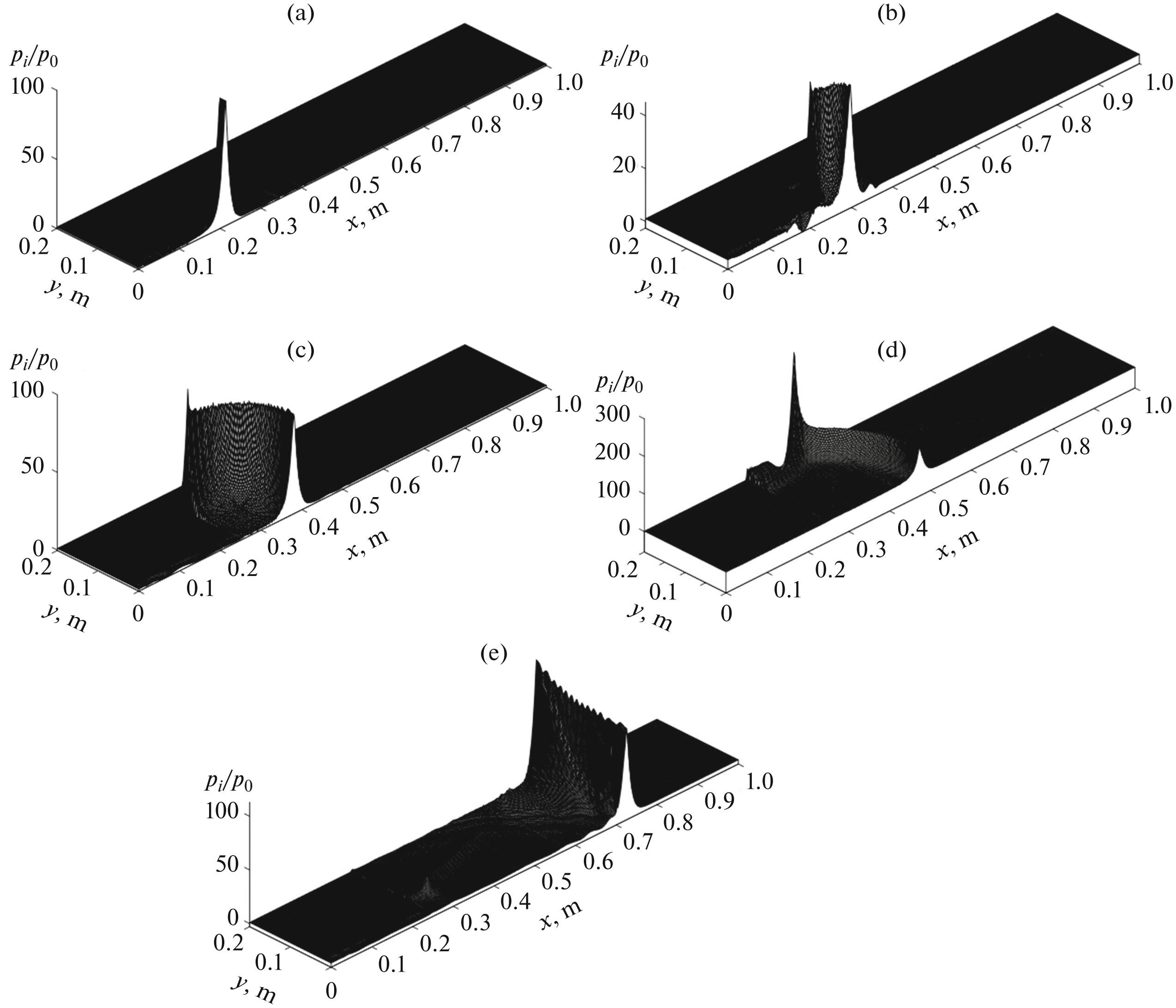

Fig. 2. Dynamics and breakdown of detonation wave in suddently expanding channel for time instants (a) 0.25, (b) 0.35, (c) 0.45, (d) 0.55, and (e) 0.80 ms. Parameters of calculation domain: $L_1 = 0.25$ m, $L = 0.5$ m, $H_1 = 0.015$ m, $H_2 = 0.175$ m; parameters of initial pulse: $\Delta u_0 = 10$ m/s $t_* = 36$ μs. Parameters of bubbly liquid: liquid, water glycerin mixture with glycerin volume content of 50%, $\rho_l^0 = 1130$ kg/m^3, $\nu_l = 6 \times 10^{-6}$ m^2/s, $c_l = 3.3$ kJ/(kg K), $\lambda_l = 0.42$ W/(m K), $C_l = 1700$ m/s, $T_0 = 293$ K; gas, acetylene–oxygen stoichiometric mixture: $\alpha_{go} = 0.005$, $a = 1.25$ mm, $\rho_g^0 = 1.26$ kg/m^3, $\lambda_g = 2.49 \times 10^{-2}$ W/(m K), $\gamma = 1.35$, $c_g = 1.14$ kJ/(kg K) $T_* = 1000$ K, $\Delta T = 3200$ K. Steps of numerical scheme of integration over time and coordinate, respectively: $\tau = 0.01$ μs, $h = 0.1$ mm.

where Δu_0 is the velocity amplitude and t_* is the characteristic pulse length. At the boundaries of the computation domain, the conditions on a rigid wall are taken, i.e., the normal velocity component is zero.

CALCULATION RESULTS

Figure 2 shows the pressure diagrams for the dynamics of a detonation wave (DW) and its entry into the expanding zone. The pressure diagrams correspond to the following time instants: (a) 0.25, (b) 0.35, (c) 0.45, (d) 0.55, and (e) 0.8 ms. As can be seen from Fig. 2a, corresponding to 0.25 ms, a detonation wave with an amplitude of about 90 atm is initiated and propagates in the narrow part of the channel under the action of the hard striker. From the diagrams corresponding to 0.35 (Fig. 2b) and 0.45 (Fig. 2c) ms, we can see how the DW is transformed at the exit from the narrow part of the channel to the wide part. Note that the front of the DW after exiting the expanding zone has a circular shape; i.e., the DW propagates uniformly both in the longitudinal and transverse directions.

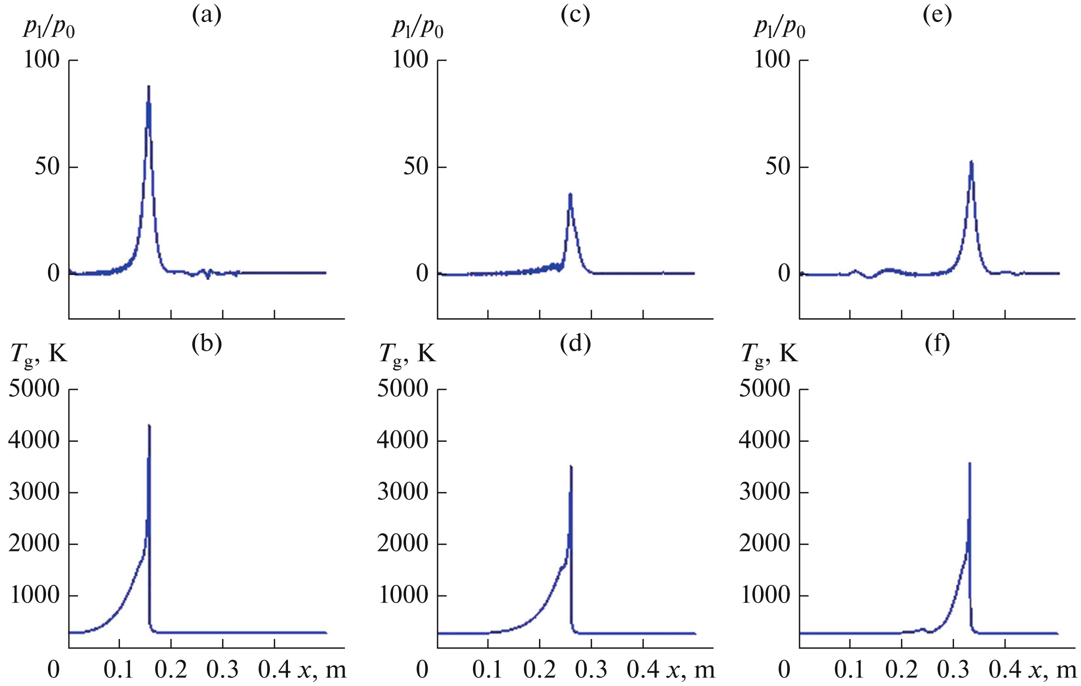

Fig. 3. Pressure and temperature diagrams for time instants: (a) and (b) 0.2, (c) and (d) 0.3, (d) and (e) 0.4 ms.

From the pressure diagram corresponding to 0.55 ms (Fig. 2d), it follows that the DW acts on the wall with an amplitude of about 30 MPa. With further evolution of the DW in the wide part of the channel, its front becomes planar, illustrated in Fig. 2d, corresponding to 0.8 ms. The propagation velocity of detonation is the most important (in addition to pressure) characteristic of DWs. The DW velocity in the narrow part of the channel is about 1100 m/s, a value close to that determined experimentally [8, 9]. In the interval ($0.25 < x < 0.45$ m), the DW velocity is about 700 m/s, and for $x > 0.45$ m, the DW amplitude is restored to a value of 9 MPa; the velocity is about 1100 m/s.

Figure 3 shows the pressure distribution (upper panels) and temperatures (lower panels) along the x coordinate on the channel axis of symmetry at different points in time. From Figs. 3a and 3b, corresponding to 0.2 ms, it can be seen that a DW with an amplitude of about 90 atm formed along the channel axis of symmetry, while the gas temperature inside the bubbles was 4200 K. Figures 3c and 3d, corresponding to 0.3 ms, show the dynamics of the DW as it enters the wide part of the channel. It can be seen from Figs. 3d and 3e that two-dimensional scattering causes the amplitude of the DW to decrease to 40 atm, and the temperature of the gas mixture, to 3500 K. Despite the decrease in amplitude of the DW when it enters the wide part of the channel, its energy is sufficient to support detonation in this region, as can be seen from Figs. 3d and 3e, corresponding to 0.4 ms. It follows from Figs. 3d and 3e that the amplitude of the DW at this point reaches 60 atm and the gas temperature in the bubbles is 3700 K. As is clear from Fig. 2b, by 0.45 ms, the amplitude of the DW increases to 90 atm.

Figure 4 is the same as Fig. 2, except that the cross section of the narrow part is smaller, $H_1 = 0.01$ m. As seen in Fig. 4a, due to the impact of the hard striker on the resting bubbly liquid, a DW forms in the narrow part of the channel, which propagates along the x coordinate with an amplitude of about 90 atm. Then, the DW passes into the wide zone; in this case, two-dimensional scattering of the DW occurs and its amplitude decreases. For calculation parameters corresponding to Fig. 4, the amplitude of the DW decreases so much that in the wide part of the channel, the energy of this wave can no longer initiate detonation; the DW breaks down and then propagates in the bubbly liquid as a pressure wave, which further attenuates in the bubbly liquid due to heat exchange and acoustic discharge.

Figure 4b shows the pressure distribution in the calculation domain after breakdown of the DW. Clearly, the amplitude of the pressure wave in this case is about 5 atm, which is insufficient to initiate detonation in the wide part of the channel; the wave decays as it evolves.

Figure 5 shows the pressure distribution (upper panels) and temperatures (lower panels) along the x coordinate, corresponding to the channel axis of symmetry at different points in time. It is clear from Figs. 5a and 5b, corresponding to 0.2 ms, that a DW with an amplitude of about 90 atm forms along the channel axis of symmetry, while the gas temperature

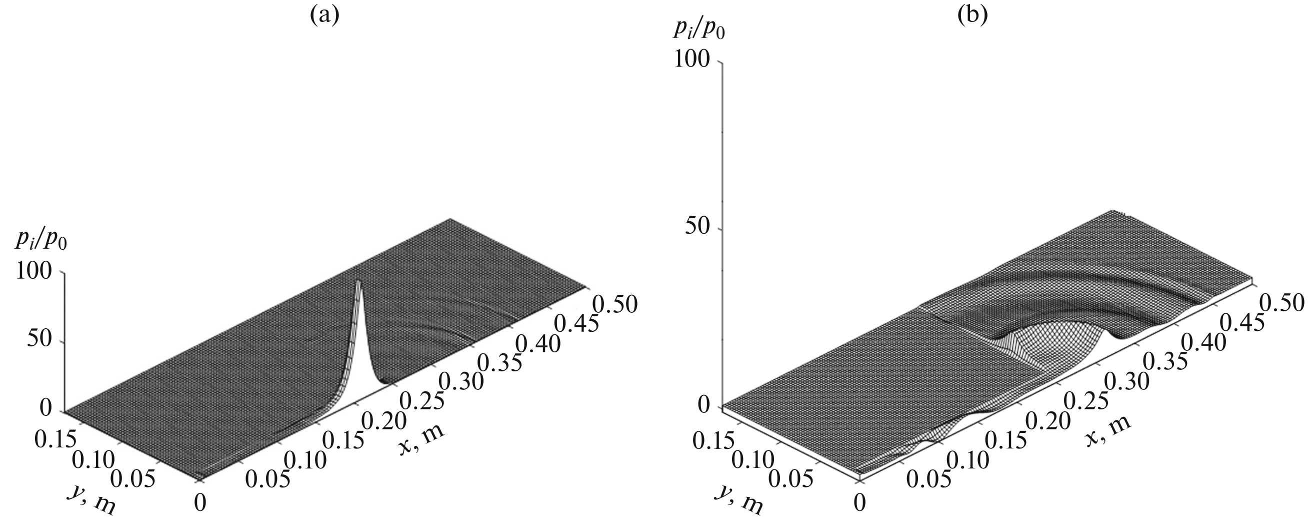

Fig. 4. Same as in Fig. 2 but $H_1 = 0.01$ m for time instants (a) 0.16 and (b) 0.3 ms. Remaining parameters are same as in Fig. 2.

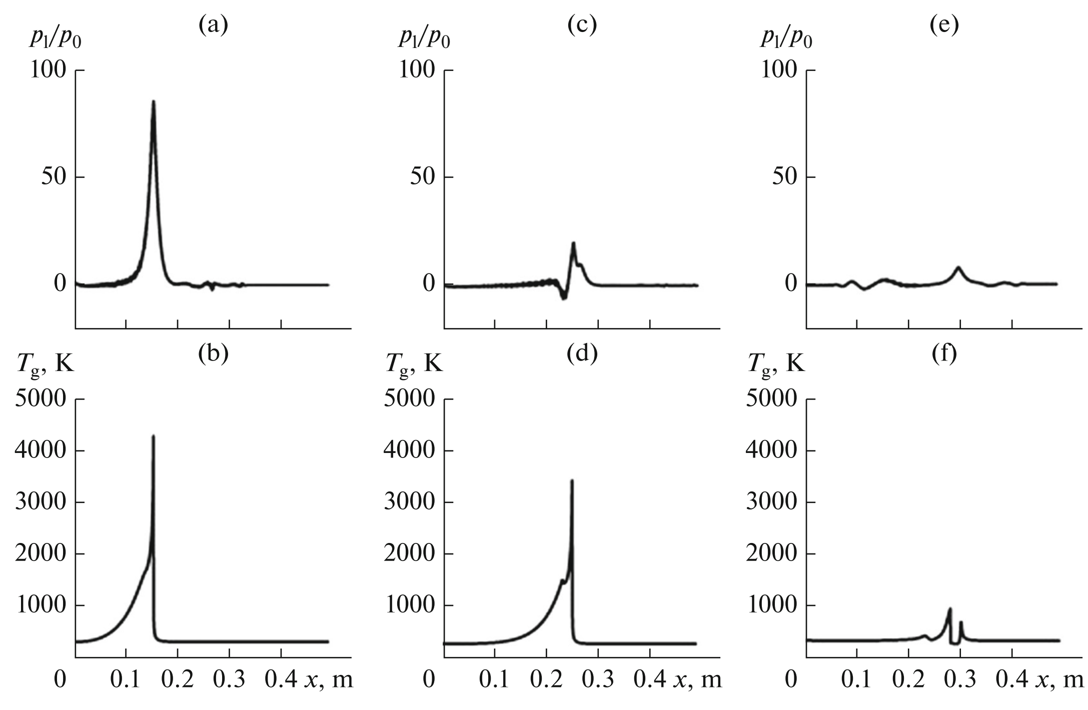

Fig. 5. Same as in Fig. 3, but $H_1 = 0.01$ m.

inside the bubbles is 4200 K. Figures 5c and 5d, corresponding to 0.3 ms, show the DW dynamics when it enters the wide part of the channel. It is clear from Figs. 5c and 5d that, owing to two-dimensional scattering, the amplitude of the DW decreases to 20 atm and the temperature of the gas mixture decreases to 3500 K. In addition, it is clear from the pressure distribution that when a wave leaves the wide part of the channel, a small-amplitude rarefaction wave appears.

In this case, the amplitude of the DW decreases so much that when it enters the wide part of the channel, its energy is insufficient to support detonation in this area and the detonation process breaks down, as seen from Figs. 5d and 5e, corresponding to 0.4 ms. It follows from Figs. 5d and 5e that the amplitude of the pressure wave propagating in a bubbly liquid after the DW has broken down is about 7 atm and the temperature of the gas in the bubbles is 700 K.

Figure 6 plots the dependence on the volume content of bubbles, all other parameters being equal, of a system with a cross section of the narrow part of the channel sufficient for the DW propagating through it

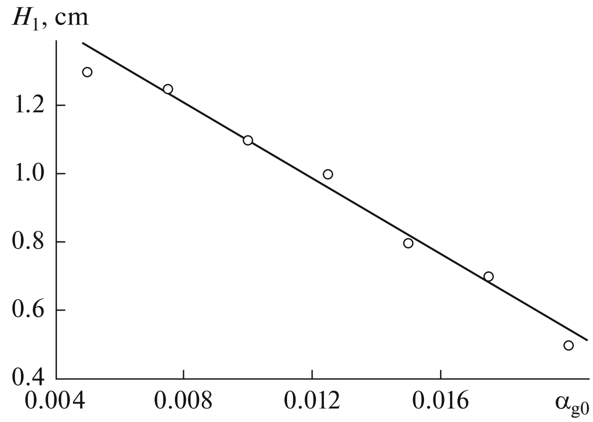

Fig. 6. Dependence of cross section size of narrow part of channel sufficient to support detonation upon exit from narrow part of channel on volume content of bubbles; other parameters are same as for Fig. 2.

to initiate detonation upon entry into the wide part. From Fig. 6 it follows that for values (α_{go}, H_1) located above the curve in Fig. 6, a DW always initiates detonation upon passing from the narrow part to the wide part, and for points (α_{go}, H_1) located below the curve, the DW breaks down upon passing from the narrow to the wide part. Numerical experiments have shown that the range of values (α_{go}, H_1) for which the detonation process breaks down is small; in addition, regardless of the H_1 value, the detonation does not break down when the ratio $\frac{H_2}{H_1} < 5$.

CONCLUSIONS

The dynamics of DWs in a bubbly liquid in suddenly expanding channels has been studied. The following features of detonation propagation in a channel with sudden expansion have been established.

(1) Two detonation propagation modes are possible when a DW passes into an expanding zone: continuous propagation and attenuation.

(2) The transition from one detonation mode to another depends not only on the cross section of the narrow part of the channel, but also on the volume content of bubbles.

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