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OCEAN ACOUSTICS. HYDROACOUSTICS

On the Method of Source Images for the Wedge Problem Solution in Ocean Acoustics: Some Corrections and Appendices[1]

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Abstract—In this study the method of source images for the problem of sound propagation in a penetrable wedge [G. Deane and M. Buckingham, J. Acoust. Soc. Am. 93 (1993) 1319–1328] is revisited. This solution is very important three-dimensional (3D) benchmark in computational underwater acoustics, since a wedge bounded from above by the sea surface and overlying a sloping penetrable bottom is the simplest model of a shallow-sea waveguide near the coastline. The corrected formulae for the positions of the source images and bottom images are presented together with the explanation of their derivation. The problem of branch choice in the reflection coefficient is thoroughly discussed, and the corresponding explicit formulae are given. In addition, numerical validation of the proposed branch choice schemes and the resulting wedge problem solutions are presented. Finally, source images solution is computed for a series of examples with different ratios of shear and bulk moduli in the bottom. The interplay between the acoustic-elastic waves coupling and the horizontal refraction in the wedge is demonstrated.

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INTRODUCTION

Significant advances in the performance of modern computers achieved over past two decades boosted the development of the numerical three-dimensional (3D) sound propagation methods in underwater acoustics. Although direct finite-difference and finite-element methods are not yet capable of handling full-scale 3D problems, various approximate techniques such as 3D parabolic equations (PEs) are remarkably successful in this field. At the same time, any approximation or numerical technique requires some reliable benchmarks to adjust its parameters and estimate its accuracy. For instance, in the case of 3D PEs it is always necessary to understand which terms of the square root approximation are crucial for handling various propagation effects (e.g. horizontal refraction, mode coupling, etc), and which of them can be neglected. Such in-depth understanding can be drawn from the comparison of the numerical results with the analytical solutions of some 3D propagation problems. Such analytical benchmark solutions are quite scarce, and one of the most important among them is the solution of the 3D wedge problem by the method of source images [1–3] proposed by Deane and Buckingham back in 1993.

The basic idea of this method is the decomposition of the total sound field into a sum of contributions from a series of source images. Each source image corresponds to the waves that were emitted by the source and underwent a certain number of interactions with the wedge boundaries [1], i.e. the surface and the bottom (see the next section).

As was mentioned above, the main application area of the source images solution is the validation of the numerical 3D models for sound propagation in ocean waveguides [4]. So far, the validations have been carried out mainly for models with the fluid bottom [5, 6]. It is widely accepted however that in the near future a benchmark solution for 3D seismo-acoustic propagation problems will be in demand, once the models and codes for 3D seismo-acoustic fields modeling are developed and implemented.

In this study, we revisit the solution of Deane and Buckingham in an attempt to clarify some points of their paper [1] and introduce some minor corrections

[1] The article is published in the original.

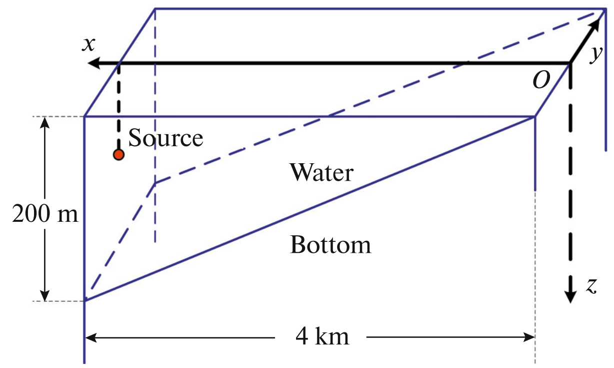

Fig. 1. Geometry of the wedge-like waveguide.

to their formulae. More precisely, we derive corrected formulae for angular coordinates of source images and inclination angles of bottom images, and provide a clear and detailed explanation of the branch choice in the reflection coefficient (which was not presented in the original study) [1]. Consistency of the proposed branch choice scheme and the accuracy of the resulting solution is validated by direct comparison against reference solutions. Furthermore, we compute the "source image solution" for a series of examples where acoustic wedges overly elastic bottom with different values of shear waves (S-waves) velocity. Effects of both bottom elasticity and horizontal refraction on the interference structure of the sound fields are presented and discussed.

PROBLEM STATEMENT

Consider a shallow-water waveguide shown in Fig. 1. It is formed by a wedge-like water layer overlying a penetrable bottom and bounded from above by the sea surface. The wedge apex angle is denoted by α_w. The sound speed and density in the water column are c_1 and ρ_1, while the respective parameters for the bottom are c_2 and ρ_2. The attenuation at the bottom is assumed to be β_2 decibels per wavelength. In the case of an elastic bottom we also introduce the S-wave speed c_{2s} and the corresponding attenuation β_{2s}. Hereafter we discuss the problem of the computation of acoustic field produced by a time-harmonic point source of frequency f located inside the water column of this waveguide (referred to as penetrable wedge).

Although this problem looks very simple, it exhibits many interesting and complicated sound propagation effects, including but not limited to, mode coupling, horizontal refraction, and diffraction at the wedge apex. The accurate simulation of all propagation features, that arise in this problem, requires extremely sophisticated mathematical technique [12]. To certain extent, this problem is a compendium of major challenges that one can encounter in the computational underwater acoustics.

Wave diffraction at the wedge apex is arguably the most difficult point of the wedge problem solution, especially in the case of very small apex angle. Much to our relief, the contribution of the diffracted wave is negligible on the typical scale of underwater acoustics (i.e. in the case when the apex is very far from the source, and the apex angle is very small).

According to Deane and Buckingham [1], the acoustical field in this case can be represented as a sum over the so-called source images $S_{n_b,l}$ numbered by the two indices (see explanation in the next section). The field due to the source image $S_{n_b,l}$ is denoted by $p_{n_b,l}$, and the total field inside the wedge can be expressed as

$$p(x,y,z) = p_{0,-1}(x,y,z) + p_{0,2}(x,y,z) + \sum_{n_b=1}^{N}\sum_{l} p_{n_b,l}(x,y,z). \tag{1}$$

The contribution of each source image can be written in the form of the plane wave decomposition as

$$p_{n_b,l} = (-1)^{n_s}\frac{i}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\prod_{b=1}^{n_b}V(\varphi_b)\right] \times \frac{\exp(i\mathbf{k}\cdot\mathbf{R})}{k_z}dk_x dk_y, \tag{2}$$

where n_s is the number of surface reflections and is determined by the values of n_b and l; the product term in the square brackets represents the total reflection coefficient associated with this particular source image; $\mathbf{R} = (x_r, y_r, z_r)$ is the position of the receiver in a local coordinate system (see Fig. 3) which originates from this specified source image; $\mathbf{k} = (k_x, k_y, k_z)$ is the wavenumber vector, with components k_x, k_y and k_z in the x_r, y_r, and z_r directions respectively and satisfying $k_z = (k^2 - k_x^2 - k_y^2)^{1/2}$; $k = \omega/c_1$ and ω is the angular frequency of the source. For the case $z_r \geq 0$, we require $\mathrm{Im}(k_z) \geq 0$ to ensure that the plane wave components satisfy the radiation condition at infinity, and vice versa.

With the transformations

$$\begin{aligned} k_x &= k\sin\theta\cos\phi, & x_r &= R\sin\zeta\cos\xi,\\ k_y &= k\sin\theta\sin\phi, & y_r &= R\sin\zeta\sin\xi,\\ k_z &= k\cos\phi, & z_r &= R\cos\zeta, \end{aligned} \tag{3}$$

Eq. (2) can be rewritten in a form that is more practical for numerical implementation:

$$p_{n_b l} = (-1)^{n_s}\frac{ik}{2\pi}\int_{0}^{\pi/2-i\infty}\exp[i\Omega_z(\theta)]\sin\theta \times \int_0^{2\pi}\left[\prod_{b=1}^{n_b}V(\varphi_b)\right]\exp[i\Omega(\theta)\cos(\phi-\xi)]\,d\theta d\phi, \tag{4}$$