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ACOUSTICAL SIGNAL PROCESSING
AND COMPUTER SIMULATION

A Method for Simultaneous Reconstruction of Temperature and Concentration Distribution in Gas Mixtures Based on Acoustic Tomography[1]

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Abstract—The acquisition of the concentration and temperature distributions in a mixed gas system plays a crucial role in real industrial applications. Different from comment measurement method, in this paper an acoustic tomography (AT) based method is developed for the simultaneous reconstruction of the concentration and temperature distributions. The proposed method includes two stages. In the first stage, the correlations of the acoustic velocity and the intensity attenuation with the temperature and concentration are investigated. Then the approach that can simultaneously compute temperature and concentration is proposed. In the second stage, in order to achieve 2D tomography, the stochastic inversion method for solving the inverse problem is developed. Numerical simulations are implemented to validate the feasibility of the proposed reconstruction method.

Keywords: acoustic tomography, relaxation attenuation, temperature distribution measurement, concentration distribution measurement, reconstruction method

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1. INTRODUCTION

Acquiring the concentration and temperature distributions in a mixed gas system plays an important role in real industrial applications. With the distinct advantages, including non-invasive sensing, high safety and low cost, acoustic tomography (AT) is considered as a promising visualization measurement method [1–3]. Currently, most applications of the AT methods focus on a single physical field measurement, but the simultaneous reconstruction of the temperature and concentration distribution in a two-dimensional domain is absent. With the increasing requirements of the multi-field measurement, seeking a reliable method to achieve the simultaneous reconstruction of the temperature and concentration distributions remains a crucial issue.

In this paper, the AT method is used to simultaneously reconstruct the temperature and concentration distributions of a gas field. Acoustic speed and acoustic attenuation, which are related to the temperature and concentration distributions, are chosen as the measurement parameters. The relationship between the sound speed and the physical quantities of a gas mixture can be formulated as [4]

$$c^2 = \gamma_{\mathrm{mix}} RT/M_{\mathrm{mix}}, \tag{1}$$

where γ_{mix} represents the mole fraction weighted ratio of specific heats of the mixture; T is the temperature; R stands for the universal gas constant and M_{mix} defines the molecular weight of the mixture.

It is challenging to establish the connection of the sound attenuation with the temperature and the concentration. Acoustic attenuation is derived from a series of mechanisms [5]. Classical attenuation caused by thermal conduction and viscosity, diffusion of the mixed gas, as well as molecular relaxation attenuation which is the main aspects discussed in this paper. Classical acoustic attenuation is calculated according to the known formulas, the diffusion of the gas mixture also can be computed using existing theories [6–7], and the relaxation attenuation at the molecular scale is complicated and has been intensively studied. Based on the quantum mechanics, Landau and Teller [8] proposed a theoretical approach describing the relationship of the relaxation time with density and temperature in a pure diatomic gas. The theory was extended to a mixture of two kinds of gases by Schwarz, Slawsky and Herzfeld (SSH) using the concept of the complex interactions of the multiple molecules [9]. In recent years, Dain and Lueptow (DL) improved the SSH theory and extended it to multicomponent or polyatomic mixtures at room temperatures based on the molecular constants [10]. Petculescu and Zhu [11–13] applied the theory to inves-

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tigate the relations between the relaxation attenuation and gas composition. In Ejakov's work [14] the attenuation from the model was compared to the measured values for a variety of gases, including air, oxygen, methane, hydrogen and mixtures of oxygen/nitrogen, methane/nitrogen, carbon dioxide/nitrogen, and hydrogen/nitrogen.

In most of the existing literatures, temperature is assumed to be a constant in order to establish the relationship between the sound attenuation and the acoustic transmission parameters. Nevertheless, there inevitably exists temperature fluctuation in some cases, and thus the hypothesis of the constant temperature may not be appropriate. This paper aims to develop a reliable method to simultaneously reconstruct temperature and concentration distributions of a gas mixture. The main contributions of this work can be outlined as follows.

1—Unlike the common single physical field reconstruction, in this paper a reconstruction method that can achieve the simultaneous reconstruction of temperature and concentration is proposed.

2—With the considerations of the inaccurate approximate properties of the model deviations and measurement error, the SI method is developed for solving the proposed inverse problem. Numerical simulations are implemented to assess the feasibility of the improved SI method.

The rest of this paper is structured as follows. Theory background on the acoustic attenuation model is concisely introduced in Section 2, and a new method is proposed to simultaneously reconstruct temperature and concentration. In section 3 the SI method is developed to achieve the simultaneous reconstruction of temperature and concentration distributions. Numerical simulations are described in Section 4. Finally, Section 5 summarizes the main conclusions.

2. THEORY

In this section, theoretical background on simultaneous reconstruction of temperature and concentration is proposed. Firstly, the dependences of classical and relaxation acoustic attenuation on concentration and temperature are deduced. Secondly, according to the coupling effect of the temperature and concentration on sound speed and attenuation, a method that can achieve the simultaneous reconstruction of the temperature and concentration is proposed.

2.1. Theoretical Background

Total attenuation of acoustic wave includes classical attenuation and relaxation attenuation. Based on the effect of gas viscosity and thermal conductivity, the classical attenuation can be computed by [6],

$$\alpha_{\text{class}} = \frac{2\pi^2 f^2}{\rho_0 c^3}\left[\frac{4}{3}\eta + (\gamma - 1)\frac{\kappa}{c_p}\right], \qquad (2)$$

where η is the gas viscosity; κ represents the thermal conductivity; c_p stands for the isobaric specific heat; γ is the mole fraction weighted ratio of specific heat; ρ_0 represents the density and f is the frequency of the acoustic wave.

In this work, a case with a negligible pressure fluctuation in a fixed volume was chosen to study the correlations of the relaxation attenuation and the mixed gas parameters. Thus the density of the mixture depends mainly on temperature and gas compositions according to the ideal gas equation $pM_{\text{mix}} = \rho RT$.

In Eq. (2), thermophysical parameters c_p, η and κ should be determined to calculate classical attenuation. (1) Isobaric molar specific heat, $c_p = a + b(T - 273.15)$ and coefficients a and b are dependent on the type of gas [15]. (2) Viscosity $\eta = 10^{-8} \times T_c^{-1/6} P_c^{2/3} M^{1/2} Z_c^m a(bT_r + c)^n$, where T_c, P_c and z_c stand for critical temperature, critical pressure and compressibility factor, respectively; T_r represents the reduced temperature and parameters a, b, c, m and n are dependent on the categories of gas. (3) Under the condition of the low pressure and medium pressure, variable κ can be computed by $\kappa = a + bT + cT^2 + dT^3$, where a, b, c and d can be acquired in [16]. Under the condition of a small fluctuation of pressure, all thermophysical parameters c_p, η and κ rely mainly on temperature and types of gas. As a result, we can find the relationship between the classical attenuation with the temperature and concentration of gas.

The relaxation attenuation related to molecular relaxation in a gas mixture is hard to predict. The energy of a molecule can be divided into two parts: internal energy and external energy, which remain in equilibrium without stimulation. When an acoustic wave passes through, the equilibrium is broken. Then molecules tend to a dynamic equilibrium by energy exchange between internal and external degrees of freedom. During this process, a fraction of the translational kinetic energy of the molecules is transferred into molecular rotation and vibration, and time required to recover from this unstable state is called relaxation time. The relaxation acoustic attenuation in a mixture is strongly dependent on the concentrations of the gas mixtures.

The propagation of sound in an excitable gas mixture is ultimately characterized by the effective wave number, which can be formulated as [17]:

$$k^2 = k_0^2 \frac{C_V^0 + \sum_{i=1}^{n} \alpha_i C_i^{\text{vib}}(\Gamma_i - 1)}{C_p^0 + \sum_{i=1}^{n} \alpha_i C_i^{\text{vib}}(\Gamma_i - 1)}, \qquad (3)$$

where α_i represents the mole fraction of each constituent in the mixture; C_V^0 and C_0^p are the translational specific heat at constant volume and constant pressure; k_0 stands for the static value of the wave number; $\Gamma_i = \Delta T_i^{\text{vib}} / \Delta T_i$ defines the temperature fluctuation ratio and C_i^{vib} represents the vibrational specific heat of species i; ΔT_i^{vib} and ΔT_i are the fluctuations of the vibrational and translational temperatures, respectively.

Note that the effective wave number is also dependent mainly on temperature and concentration. The acoustic attenuation is the imaginary part of the effective wave number k. Thus, the relationship between the acoustic attenuation and the concentration of the gas mixture has been ascertained.

Consequently, both of the dependence of sound speed and attenuation on temperature and concentration are obtained. Two coupling models are built, one is the dependence of sound speed on temperature and concentration (see Fig. 1a). The other one is the dependence of acoustical attenuation on temperature and concentration (see Fig. 1b).

2.2. Simultaneous Reconstruction Method of Dual-physical Parameter

Most of the existing research focuses on a single parameter measurement, such as temperature or concentration, in an even or equilibrium condition. This section introduces a method that can simultaneously reconstruct temperature and concentration in a non-equilibrium environment.

According to Eqs. (1)–(3) and Section 2.1, an example is used to introduce the simultaneous reconstruction method of the dual-physics field. CO–N_2–O_2–H_2O is chosen as the gas mixture, in which $N_2 : O_2 : H_2O$ is 78 : 21 : 1. Assuming that the temperature distribution and CO concentration in CO–N_2–O_2–H_2O mixture are unknown, and the sound speed measured is $c_0 = 498$ m/s. As shown in Fig. 1a, a horizontal slice through the surface at the measured sound speed c_0 results in a curve on the temperature-concentration plane. Any of the temperature-concentration combinations defined along this curve has the same sound speed, temperature and concentration cannot be uniquely determined. Likewise, a similar curve can be obtained using the measured acoustic attenuation data $\alpha_0 = 1.5 \times 10^{-3}$ m^{-1} (see Fig. 1b). Then, by making the two curves project onto one temperature-concentration plane, the temperature of the gas mixture and the concentration of CO can be obtained from the position of the point of intersection (see Fig. 1c). Therefore, temperature and concentration can be determined through the measurement values of acoustical attenuation and speed.

In the case of a 2D domain, we assume that the temperature and concentration distributions are unknown and inhomogeneous. The domain is divided into a series of small square cells and the physical state in every cell is seen as the same. Following the AT theory [18], the sound speed and acoustic attenuation can be reconstructed by a series of measurements of travel-time and intensity attenuation of the acoustic signal. Figure 2a shows the arrangement of the acoustic transducers.

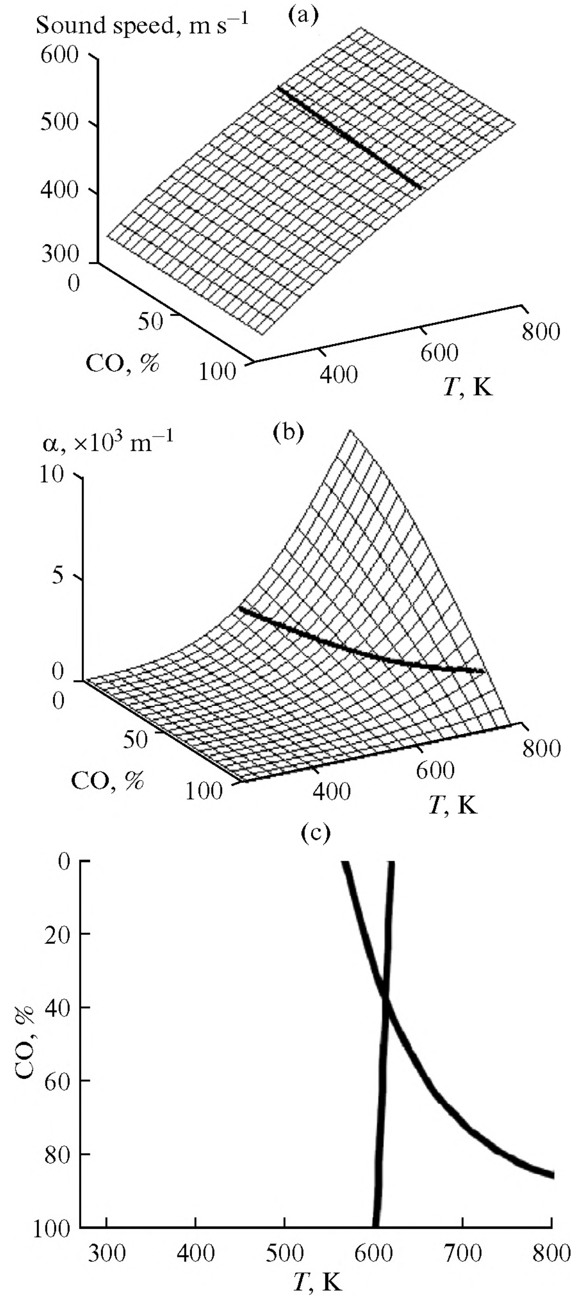

Fig. 1. (a) Dependence of the sound speed on concentration of CO and temperature of the gas mixture, (b) dependence of the acoustical attenuation on concentration of CO and temperature of the gas mixture, (c) intersection of the two curves projected onto one coordinate plane.

3. RECONSTRUCTION ALGORITHM

The relations between measured acoustic data and state parameters of the gas mixtures have been dis-

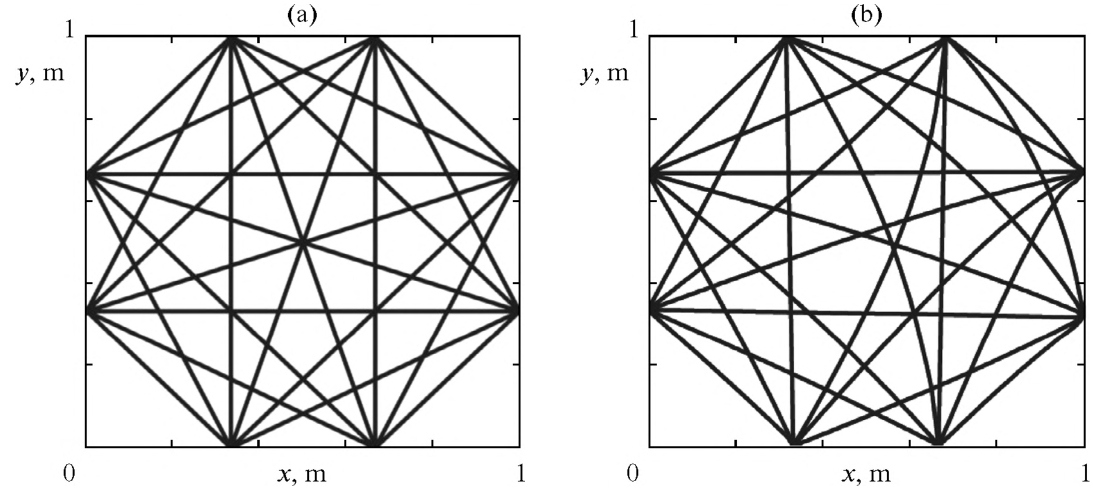

Fig. 2. Acoustic paths: (a) straight lines, (b) actual lines.

cussed in the previous sections, and how to determine the sound speed and attenuation of intensity in each pixel is another problem that need to be considered. Section 3.1 introduces the SI algorithm according to the idea of the Wiener filtering [19]. In Section 3.2 the SI method is developed for the improvement of the reconstruction quality. In Section 3.3 an alternating iteration procedure is proposed to achieve the simultaneous reconstruction of dual-physics fields.

3.1. Stochastic Inverse Method

The inverse problem in this work is non-linear and underdetermined, and numerous methods are available for solving this problem [20–22]. Among the various kinds of methods, the SI method is suitable to solve underdetermined inverse problems [23]. In the SI algorithm the temperature and concentration of the gas mixture is assumed to be random fluctuations with the given spatial correlation functions, then the fields with minimized mean square error is sought.

In this work, temperature, concentration, sound speed and acoustic attenuation of the gas mixtures are formulated as a superposition of the spatial average values and the fluctuation values:

$$\begin{gathered} T(\mathbf{r}) = T_0 + T'(\mathbf{r}), \quad \alpha(\mathbf{r}) = \alpha_0 + \alpha'(\mathbf{r}), \\ c(\mathbf{r}) = c_0 + c'(\mathbf{r}), \quad att(\mathbf{r}) = att_0 + att'(\mathbf{r}), \end{gathered} \tag{4}$$

where $\mathbf{r} = (r_1;\ldots r_j\ldots;r_J)$ is a position vector in a two-dimensional reconstruction domain; J is the number of the pixel; T_0, α_0, c_0 and att_0 represent the spatial average values of temperature, concentration, sound speed and acoustic attenuation; $T'(\mathbf{r})$, $\alpha'(\mathbf{r})$, $c'(\mathbf{r})$ and $att'(\mathbf{r})$ stand for the fluctuation values, respectively.

The first task is to calculate mean values of T_0 and α_0. The procedure can be summarized as follows: assuming that the field has no fluctuations and given the travel times and intensity attenuation, then c_0 and att_0 can be calculated by the least squares method. Based on Figs. 1a and 1b, the mean values of temperature and concentration distributions (T_0 and α_0) can be obtained.

The travel-time τ_i and attenuation of intensity att_i of the sound wave propagating along path L_i can be formulated as:

$$\tau_i = \varepsilon_i + \int_{L_i} \frac{1}{c_0 + c'(\mathbf{r})} ds, \tag{5}$$

$$att_i = \sigma_i + \int_{L_i} \left(att_0 + att'(\mathbf{r})\right) ds, \tag{6}$$

where ε_i and σ_i are the errors of τ_i and att_i, which contain the errors in travel time measurements and transducer positions.

If the absolute values of fluctuations in the sound speed are much smaller than the spatial average value c_0, such as in atmosphere measurement, Eq. (5) can be approximated as the first order expansion of the fluctuations, which can be formulated as [23]

$$\tau_i = \frac{L_i}{c_0} - \frac{1}{c_0^2} \int_{L_i} c'(\mathbf{r}) ds. \tag{7}$$

Then, define column vectors $\mathbf{d}_1$ and $\mathbf{d}_2$ that are related to travel time and sound attenuation as:

$$\begin{aligned} \mathbf{d}_1 &= [L_1c_0 - \tau_1c_0^2 + \varepsilon_1; L_2c_0 - \tau_2c_0^2 \\ &\quad + \varepsilon_2;\ldots;L_Ic_0 - \tau_Ic_0^2 + \varepsilon_I], \end{aligned} \tag{8}$$

$$\begin{aligned} \mathbf{d}_2 &= [att_1 - L_1\alpha_0 + \sigma_1; att_2 - L_2\alpha_0 \\ &\quad + \sigma_2;\ldots;att_I - L_I\alpha_0 + \sigma_I], \end{aligned} \tag{9}$$

where I is the number of the acoustic paths. Combining Eqs. (6)–(9), the elements of data column vector $\mathbf{d}_1$ and $\mathbf{d}_2$ without measurement noises can be written as:

$$\mathbf{d}_1 = \left[\int_{L_1} c'(\mathbf{r}) ds; \int_{L_2} c'(\mathbf{r}) ds;\ldots; \int_{L_I} c'(\mathbf{r}) ds \right], \tag{10}$$

$$\mathbf{d}_2 = \left[\int_{L_1} \alpha att'(\mathbf{r})ds; \int_{L_2} \alpha att'(\mathbf{r})ds; \ldots; \int_{L_I} \alpha att'(\mathbf{r})ds \right]. \quad (11)$$

According to the measurement information, the problem is to reconstruct the fluctuations of $c'(\mathbf{r})$ and $att'(\mathbf{r})$, which can be expressed as two vectors $\mathbf{m}_1$ and $\mathbf{m}_2$:

$$\mathbf{m}_1 = [c'(r_1); c'(r_2); \ldots; c'(r_J)], \quad (12)$$

$$\mathbf{m}_2 = [att'(r_1); att'(r_2); \ldots; att'(r_J)]. \quad (13)$$

Thus, the solution of variables $\mathbf{m}_1$ and $\mathbf{m}_2$ can be rewritten as a matrix equation:

$$\mathbf{m}_1 = \mathbf{A}_1\mathbf{d}_1, \quad (14)$$

$$\mathbf{m}_2 = \mathbf{A}_2\mathbf{d}_2, \quad (15)$$

where $\mathbf{d}_1$ and $\mathbf{d}_2$ are the measurement information; matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ are optimal stochastic inverse operators, the core work in original SI algorithm is to establish $\mathbf{A}_1$ and $\mathbf{A}_2$ which can directly give the unknown parameters $\mathbf{m}_1$ and $\mathbf{m}_2$. The principle of calculating of the optimal stochastic inverse operators is to minimize the mean square errors [23, 24]. For simplicity, $\mathbf{A}$ is used to represent $\mathbf{A}_1$ or $\mathbf{A}_2$, $\mathbf{d}$ denotes measured date vector $\mathbf{d}_1$ or $\mathbf{d}_2$ and $\mathbf{m}$ is the solution of variable $\mathbf{m}_1$ or $\mathbf{m}_2$. The inverse operator $\mathbf{A}$ can be written as

$$\mathbf{A} = \mathbf{R}_{\mathbf{md}}\mathbf{R}_{\mathbf{dd}}^{-1}, \quad (16)$$

where $\mathbf{R}_{\mathbf{md}} \equiv \langle \mathbf{md}^T \rangle$ and $\mathbf{R}_{\mathbf{dd}} \equiv \langle \mathbf{dd}^T \rangle$ are the model-data and data-data covariance matrices. In the case of AT, one can use the relationship between $\mathbf{d}$ and unknown field $\mathbf{m}$ given by Eq. (10) and (11) to obtain the expression for the model-data covariance matrix $\mathbf{R}_{\mathbf{md}}$:

$$[\mathbf{R}_{\mathbf{md}}]_{ji} = \langle m_j d_i \rangle$$
$$= \begin{cases} \left\langle c'(r_j), \int_{L_i} c'(\mathbf{r})ds \right\rangle \\ \left\langle att'(r_j), \int_{L_i} att'(\mathbf{r})ds \right\rangle \end{cases} = \begin{cases} \int_{L_i} \langle c'(r_j), c'(\mathbf{r}) \rangle ds \\ \int_{L_i} \langle att'(r_j), att'(\mathbf{r}) \rangle ds \end{cases} \quad (17)$$
$$= \begin{cases} \int_{L_i} B_{c'c'}(r_j, \mathbf{r})ds & \text{for sound speed,} \\ \int_{L_i} B_{att'att'}(r_j, \mathbf{r})ds & \text{for acoustic attenuation,} \end{cases}$$

where $B_{c'c'}$ and $B_{att'att'}$ are the spatial covariance functions of the sound speed and acoustic attenuation. r_j is the chosen spatial points within the reconstructed area, and these points stay fixed during the integration.

Similarly, an expression for the covariance matric $\mathbf{R}_{\mathbf{dd}}$ is given by

$$[\mathbf{R}_{\mathbf{dd}}]_{ip} = \langle d_i d_p \rangle$$
$$= \begin{cases} \left\langle \int_{L_i} c'(\mathbf{r})ds, \int_{L_p} c'(\mathbf{r}')ds \right\rangle \\ \left\langle \int_{L_i} att'(\mathbf{r})ds, \int_{L_p} att'(\mathbf{r}')ds \right\rangle \end{cases} \quad (18)$$
$$= \begin{cases} \int_{L_i}\int_{L_p} B_{c'c'}(\mathbf{r}, \mathbf{r}')dsds & \text{for sound speed,} \\ \int_{L_i}\int_{L_p} B_{att'att'}(\mathbf{r}, \mathbf{r}')dsds & \text{for acoustic attenuation,} \end{cases}$$

where $i, p = 1, 2, \ldots, I$, $\mathbf{r} \in L_i$, $\mathbf{r}' \in L_p$.

In this work the Gaussian covariance function is employed to describe the spatial covariance of the sound speed variation $B_{c'c'}$, and the acoustic attenuation variation $B_{att'att'}$, which can be represented as [24]

$$B_{c',c'}(\mathbf{r}, \mathbf{r}') = \sigma_c^2 \exp\left(-\frac{(\mathbf{r} - \mathbf{r}')^2}{l_c^2} \right), \quad (19)$$

$$B_{att',att'}(\mathbf{r}, \mathbf{r}') = \sigma_{att}^2 \exp\left(-\frac{(\mathbf{r} - \mathbf{r}')^2}{l_{att}^2} \right), \quad (20)$$

where σ_c and σ_{att} are the standard deviations of the corresponding fields, respectively; l_c and l_{att} are the scale parameters, respectively.

3.2. Modified SI Method

It is found that the solutions from common SI method are not satisfactory when the fluctuation of temperature distributions in a measurement domain is large. In order to improve the reconstruction quality, the following modifications are proposed.

Firstly, when the SI algorithm is employed to solve the AT inverse problem, Eq. (5) is approximated as Eq. (7) by assuming that the sound speed fluctuations are much smaller than the spatial average value of c_0. This linearization assumption is feasible when the fluctuation of the temperature is small; however, it may not be appropriate when the temperature change is relatively large. In Eq. (7), with the increase of temperature fluctuation, the assumption $c_0^2 \gg c'(\mathbf{r})^2$ may be unreasonable, and thus Eq. (7) can be revised as:

$$\tau_i = \frac{L_i}{c_0} - \frac{1}{c_0^2} \int_{L_i} c'(\mathbf{r})ds + B_i, \quad (21)$$

where B_i is the model error derived from the linearization approximation of the temperature fluctuations. The model deviation B_i can be calculated by means of the alternating iteration method, which will be introduced in Section 3.3.

Secondly, the travel paths of sound are approximated by straight lines in Section 3.1, which would bring deviations when the temperature gradient is relatively large, such as in a burning furnace. Refraction will happen when sound waves pass through different temperature domain, and the acoustic paths will be bent. Ignoring the refraction will lead to errors in determining the average sound speed and the path of integration, which is crucial for the accuracy of matrix **A**.

The most complicated work in the SI method is to determine matrix **A** using spatial covariance information and locations of the transducers. One should compute the covariance functions along the acoustic path L_i, which is assumed as line at first. Once L_i bends in an uneven temperature field, we should acquire real travel path of the acoustic wave.

Based on Snell's Law, the real acoustical travel path can be calculated by the acoustic ray tracing in an inhomogeneous field [25, 26]. In our work the deployed triangles method is used to trace the actual acoustic path [26]. The main idea of obtaining the actual travel paths is to first calculate the sound speed distribution by common SI method. Then, based on deployed triangles tracing method, the actual track from the positions of emitter to receiver is computed using the rough sound speed distribution (see Fig. 2b). The revised sound paths are used to estimate matrix **A** to update the sound speed distribution and then calculate actual path again. Repeating the above process, a more accurate acoustic velocity distribution may be obtained.

3.3. Alternating Iteration Procedure

Following the above discussions, we propose an iteration scheme to simultaneously reconstruct the temperature and concentration distributions, which can be summarized as follows:

Step 1. Compute the rough velocity field according to the transducer positions and travel times. In this step the acoustic paths are assumed as straight and the velocity fluctuations are linearized (see Eq. (21) for more details), and B_i is set as zero.

Step 2. Use velocity field $c(\mathbf{r})$ obtained in Step 1 to calculate the actual acoustic path and update matrix **A**.

Step 3. Compute model error B_i based on the differentials of travel times τ_i and $\frac{L_i}{c_0} - \frac{1}{c_0^2}\int_{L_i} c'(\mathbf{r})ds.$

Step 4. Use the updated matrix **A** and model error B_i to calculate velocity field $c(\mathbf{r})$ again. Loop to Step 2 until B_i is less than a predetermined value.

Step 5. Compute acoustic attenuation in every pixel using the common SI method.

Step 6. Reconstruct temperature and concentration distributions via the method proposed in Section 2.2.

4. NUMERICAL SIMULATIONS

In this section, numerical simulations are implemented to evaluate the feasibility of the proposed reconstruction method. A square vessel with the size of 1×1 m is employed. The purpose of the simulations is to monitor the mixing process of CO and air. Eight ultrasonic transducers are arranged uniformly around the cross section of the vessel to reconstruct the temperature field and the concentration distribution of CO and air. The cross section is divided into 20×20 squares. The frequency of the acoustical transducer is chosen as 10 kHz.

The simulation include two main procedures: the forward problem and the inverse problem. According to Eqs. (1)–(3), the forward problem is to compute the travel times and intensity attenuations of each path between the ultrasonic transducers from the given fields and positions of transducers. The inverse problem is to estimate the sound speed and acoustic attenuation in every pixel and then to reconstruct the temperature and concentration fields of CO–N_2–O_2–H_2O ($N_2 : O_2 : H_2O = 78 : 21 : 1$) by the method presented in Section 2.

Two cases are selected to reflect different situations of gas mixtures.

Case 1. Temperature and concentration fields vary within a relatively wide range, the temperature changes from 308 to 701 K and the concentration of CO varies in the range of 0.01 to 89%.

Case 2. Temperature and concentration fields vary within a relatively narrow range, the temperature changes from 297 to 348 K and the concentration of CO varies in the range of 0.01 to 53%.

In the numerical experiments four parameters σ_c, σ_{att}, l_c, and l_{att} should be predetermined. Once the temperature and concentration fields are given in the forward problem, these parameters can be estimated. In Case 1 $l_c = 0.15$ m, $l_{att} = 0.15$ m, $\sigma_c = 6.99$ m/s, $\sigma_{att} = 8.32 \times 10^{-6}$. However, in Case 2 the parameters are $l_c = 0.15$ m, $l_{att} = 0.15$ m, $\sigma_c = 41.08$ m/s, $\sigma_{att} = 7.3075 \times 10^{-4}$. The simulated travel time and acoustic attenuation of intensity are contaminated using Gaussian distribution random number with the covariance of 3% to simulate a real measurement environment.

Figures 3 and 4 are the reconstruction results of Cases 1 and 2, respectively. Figures 3a, 3b and 4a, 4b show the original temperature and the concentration distributions, respectively. The results of the intermediate variables, sound speed and acoustic intensity attenuation are presented in Figs. 3c, 3d and 4c, 4d. The Figs. 3e, 3i and 4e, 4i are the reconstructed temperature fields by the original SI method and the modified SI algorithm, respectively. The reconstruction results of the concentration distribution are illustrated in Figs. 3f, 3j and Fig. 4f, 4j. Figures 3g, 3k, 3h, 3l and 4g, 4k, 4h, 4l show the reconstructed images of sound speed and acoustic intensity attenuation obtained

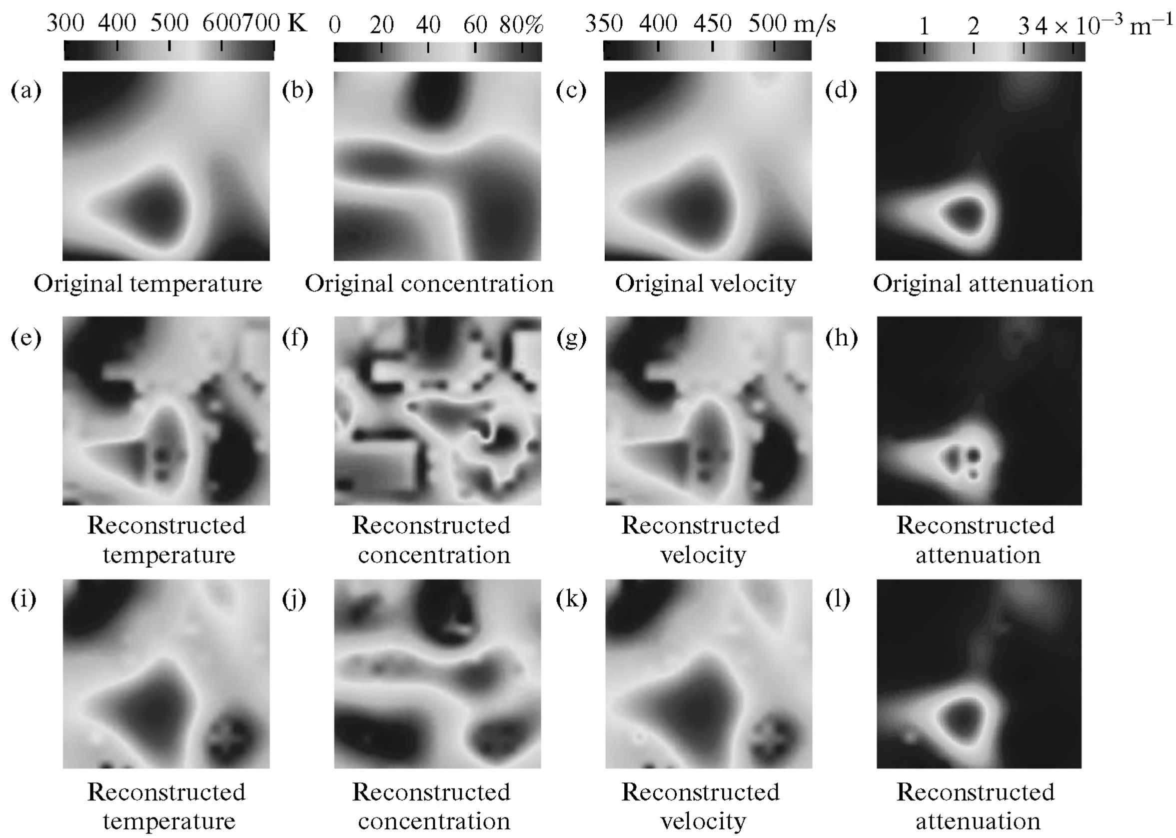

Fig. 3. Case 1: (a–d) original temperature field, concentration field, acoustic speed field, acoustic intensity attenuation field, (e–h) reconstructed temperature, concentration, velocity, intensity attenuation distributions by the original SI method, (i–l) reconstructed temperature, concentration, velocity, intensity attenuation distributions by the modified SI method.

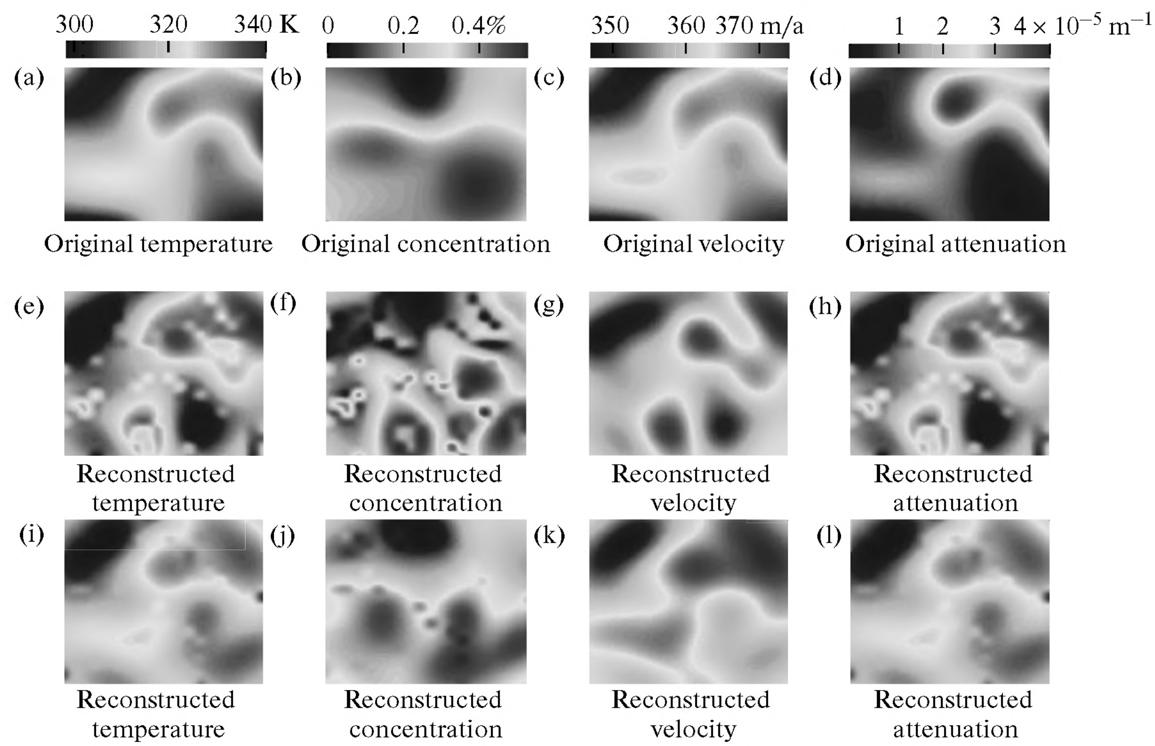

Fig. 4. Case 2: (a–d) original temperature, concentration, acoustic velocity, acoustic intensity attenuation distributions, (e–h) reconstructed temperature, concentration, velocity, intensity attenuation distributions by the original SI method, (i–l) reconstructed temperature, concentration, velocity, intensity attenuation distributions by the modified SI method.

Reconstruction errors, %

Error		Velocity	Attenuation	Temperature	Concentration
Case 1	Original SI method	5.19	15.24	7.89	36.14
	Modified SI method	3.51	12.81	4.61	32.09
Case 2	Original SI method	3.98	13.57	5.69	33.84
	Modified SI method	2.98	12.42	3.24	27.12

using the original SI method and the modified SI algorithm.

It can be found from Figs. 3 and 4 that in Case 1 the acoustic speed is increased to 525.8 m/s from 346.5 and the attenuation of acoustic energy is increased to 4.1×10^{-3} m^{-1} from 2.48×10^{-6}. In contrast, in Case 2 the sound speed changes from 347.5 to 376.8 m/s and the attenuation of acoustic energy changes from 3.83×10^{-6} to 2.48×10^{-5}.

As shown in Figs. 3 and 4, the quality of the reconstructed images by the modified SI method is improved. Images reconstructed by modified SI method are much smoother. As compared with Figs. 3e, 3f and 4e, 4f, the reconstruction results of the temperature and concentration in Case 1 is worse than Case 2. One of possible reasons is that it is hard for the common SI method to deal with the reconstruction tasks with a large variation of the temperature and concentration distributions. Meanwhile, it can be found that the distribution of concentration has a relatively smaller impact on the sound speed and attenuation in the CO-air mixture simulated in this section. Average relative molecular weight of CO is 28, and it is equal to the main component in air, N_2. According to Eq. (1), the concentration of CO has little impact on sound speed, and only changes the acoustic attenuation, which can explain the smaller impact of the concentration on speed and attenuation.

The errors of the target fields and intermediate variables are summarized in table. It can be seen that the reconstruction results from the modified SI method are improved as compared with common SI method owing to the fact that the improved SI method considers the large fluctuations of the reconstruction objects. Especially, it can be found that errors of the intermediate variables, velocity and attenuation are amplified in temperature and concentration reconstruction. One of possible reasons is that the coupling model may be inaccurate, and this issue should be further investigated in the future.

Following the above discussions, it can be concluded that the proposed method can achieve the simultaneous reconstruction of the temperature and concentration distributions.

5. CONCLUSIONS

In this paper an AT based reconstruction method has been proposed to simultaneously reconstruct the temperature and concentration distributions in a gas mixture system. The SI algorithm has been developed for solving the inverse problem, and then the temperature and concentration distributions are simultaneously reconstructed. Numerical simulation results validate the feasibility of the proposed reconstruction method.

Our work provides a new insight for acoustical measurement of the temperature and concentration distributions, which needs to be further validated by more cases and be further investigated on the respects such as numerical methods, the improvement of the reconstruction quality and the signal excitation method in the future.

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