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ACOUSTIC SIGNAL PROCESSING AND COMPUTER SIMULATION

Modeling and Effect of Distortion Product Generated by Harmonic Complex Tones[1]

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Abstract—To study deeply the effect of distortion product on auditory perception, a functional model is proposed to generate distortion products at frequencies below those of primary stimuli. The operations include calculating different power of the stimuli, low pass filtering, searching optimum weights, and summing the weighted signals across all filtering channels. The model uses simulate annealing and genetic algorithm to search the globally optimum weights. Moreover, this paper studies the effect of distortion products on pitch perception for unresolved harmonics based on the proposed model. Results find that distortion products could enhance the resolvability and temporal information of the harmonics. Thus, it is suggested to use background noise with appropriate sound levels to mask distortion products to reduce the effect on pitch perception.

Keywords: harmonic complex tone, distortion product, functional model, pitch perception, optimum weight

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1. INTRODUCTION

Many sounds in the environment are harmonic complex tones (HCTs), e.g., speech and music. When an HCT stimulates the cochlea, human auditory response is found to show not only linearity, but also nonlinearity. Thus, when two or more sinusoidal signals stimulate the cochlea simultaneously, the response contains not only the original stimuli, but also new components, which are called distortion products (DPs). Similar with other factors representing compressive nonlinearity of basilar membrane [1, 2], DPs also represent one kind of auditory nonlinearity. For two sinusoidal stimuli with frequencies of f_1 and f_2 ($f_1 < f_2$), DPs include new components at frequencies of $f_2 + f_1$, f_2–f_1, $2f_1$–f_2, $2f_2$–f_1, $3f_1$–$2f_2$, etc.

At present, there have been several psychoacoustic and electrophysiological studies on DPs. The main findings are as follows.

1—DPs at frequencies above f_1 are generally inaudible [3, 4]. On the contrary, DPs at frequencies below f_1, such as $k(f_2$–$f_1)$ and f_1–$k(f_2$–$f_1)$ (k is an arbitrary integer), are audible to human ears [4–7]. Even when the level of the primary stimuli is low or medium, the f_1–$k(f_2$–$f_1)$ DP can still be perceived [4, 7]. The sound level of f_1–$k(f_2$–$f_1)$ decreases with increasing k [4]; the maximum k is about 5 or 6 [7]. The prominent DPs at frequencies below f_1 are f_2–f_1, $2f_1$–f_2, and $3f_1$–$2f_2$ [3, 4, 6, 7]. Up to now, there have been many studies on DPs f_2–f_1 and $2f_1$–f_2 [3, 4, 6–8].

2—DP $2f_1$–f_2 is the most prominent component [4], the sound level of which is relatively independent on the stimuli level, but dependent on the frequency interval of stimuli, i.e., f_2–f_1 [3, 4, 8, 9]. When the frequency interval increases (the ratio of f_2/f_1 increases), the sound level of DP $2f_1$–f_2 will decrease. Psychoacoustic studies found that, when f_1 was 1000 Hz and f_2 was 1100 Hz, the level of DP $2f_1$–f_2 was about 14 dB below that of the primary stimuli [4].

3—For the f_2–f_1 DP, its sound level is relatively independent on frequency interval, but dependent on the level of the stimuli. The f_2–f_1 DP could be perceived only when the level of the primary stimuli is above 50 dB SL [3, 4]. Pressnitzer and Patterson [10] found that, for DPs at frequencies of 100 to 400 Hz, the f_2–f_1 DP (100 Hz) had the highest level. If there was only one pair of components in the stimuli, the level of the f_2–f_1 DP was about 20–25 dB lower than that of stimuli, which was generally consistent with the results of Goldstein [4].

4—Robels et al. [8] found that DPs on the basilar membrane of chinchilla cochlea evoked by two-tone stimuli had similar characteristics with DPs measured in psychoacoustic and electrophysiological experiments for human cochlea.

The results of the above research have provided psychoacoustic and electrophysiological support to DPs. In order to study DPs deeply, it is necessary to set

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up a functional model of DPs as a mathematical expression of auditory nonlinearity corresponding to DPs. The aim of modeling is closely relative to the mechanism of generating DPs, and relative to studying the effect of DPs on auditory pitch perception of HCTs.

2. A FUNCTIONAL MODEL GENERATING DPS

2.1. The Expression of HCT

For an HCT $x(t)$ containing M sinusoidal components, it has the form shown by Eq. (1), in which Ω_i and $A_i (i = 1, 2, 3, ..., M)$ represent the angular frequency and amplitude for each component, respectively.

$$x(t) = \sum_{i=1}^{M} A_i \cos(\Omega_i t). \quad (1)$$

2.2. Hypothesis of the Mechanism of Generating DPs

In information theory, multiplying the components of the original stimuli can generate new components. The frequencies of the latter are multiple times, difference, or sum of the frequencies of the former. The process of multiplying is defined as modulation, including self- and inter-modulation. This paper supposed a hypothesis, for which DPs were generated by the modulation of the sinusoidal components of the primary stimuli, including self- and inter-modulation by different times.

2.3. Different Power of the Stimuli

When a sound $x(t)$ stimulates the cochlea, modulation among the components of $x(t)$ occurs at the same time, which generates $x^2(t)$ shown by Eq. (2). Equation (2) has the expanded form of Eq. (3), in which DPs at frequencies of $2\Omega_i$, $\Omega_i + \Omega_j$, and $\Omega_j - \Omega_i$ can be obtained.

$$x^2(t) = \left[\sum_{i=1}^{M} A_i \cos(\Omega_i t)\right]^2, \quad (2)$$

$$x^2(t) = \sum_{i=1}^{M} \frac{A_i^2}{2}[1 + \cos(2\Omega_i t)] \\ + \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} A_i A_j [\cos((\Omega_i + \Omega_j)t) + \cos((\Omega_j - \Omega_i)t)]. \quad (3)$$

When M is 2, the DPs generated are shown by Eq. (4).

$$x^2(t) = \frac{A_1^2(1 + \cos(2\Omega_1 t))}{2} + \frac{A_2^2(1 + \cos(2\Omega_2 t))}{2} \\ + A_1 A_2 [\cos((\Omega_1 + \Omega_2)t) + \cos((\Omega_2 + \Omega_1)t)]. \quad (4)$$

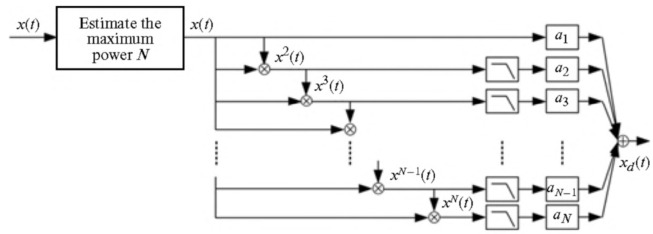

Fig. 1. The frame of the model.

If one more modulation occurs, i.e., $x^2(t)$ is multiplied by $x(t)$, $x^3(t)$ can be obtained. During the modulation, more components of DPs are generated, including $3\Omega_i$, $2\Omega_i + \Omega_j$, $2\Omega_j + \Omega_i$, $2\Omega_j - \Omega_i$, and $2\Omega_i - \Omega_j$.

Derivations of $x^4(t)$, $x^5(t)$, ..., $x^N(t)$ and their DPs can be obtained in the same way. These results with different power consist of the primary components of mathematic model.

2.4. The Structure Frame of the Functional Model

Derivations of $x(t)$, $x^2(t)$, ..., and $x^N(t)$ show that, for $x^{2k+1}(t) (k \geq 1)$, DPs at frequencies below f_1 have the forms of $2f_1 - f_2$, $3f_1 - 2f_2$, ..., and $f_1 - k(f_2 - f_1)$. For $x^{2k}(t) (k \geq 1)$, DPs at frequencies below f_1 have the forms of $f_2 - f_1$, $2(f_2 - f_1)$, ..., and $k(f_2 - f_1)$. According to the mathematical relationship between DPs and odd- and even-order power of the stimuli summarized above, this paper proposed a functional model generating DPs at frequencies below f_1 for $x(t)$, which is shown in Fig. 1.

The detail steps are as follows:

1—Estimate the maximum power N for $x(t)$.

2—Calculate $x^n(t)$, where n ranges from 2 to N.

3—Pass $x^n(t)$ $(n \in [2, N])$ through a lowpass filter, which could get rid of the high-frequency DPs at frequencies above f_1. The maximum frequency of the DPs below f_1 is $2f_1 - f_2$. Thus the cutoff frequency of the filter is set as the middle value of f_1 and $2f_1 - f_2$, which is $(3f_1 - f_2)/2$.

4—Weight $x^n(t)$ with parameter of a_n and sum as the form of Eq. (5). For $x(t)$, the weighting parameter a_1 is 1.

$$x_d(t) = \sum_{n=1}^{N} a_n x^n(t). \quad (5)$$

Please note that some of the variables in the model have to be limited, thus the output of the model can accord with the previous psychoacoustic and electrophysiology results. For example, in pure mathematics, the maximum power N could be an arbitrary integer. However, according to the previous studies [7], the appropriate N in the above model is suggested to be about 5.

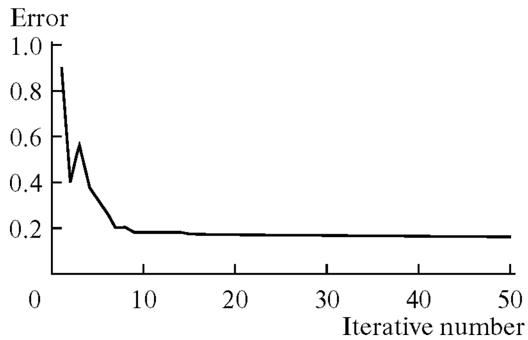

Fig. 2. Iterative error ($N = 5$).

2.5. Searching the Optimum Weights of the Functional Model

The weights of the model depend on the sound pressure levels of DPs measured by psychoacoustic experiments. Previous studies found that the perceived DPs in humans had similar characteristics with basilar membrane movement of chinchilla cochlea [8]. According to the shape of the basilar membrane movement [8], the levels of $k(f_2 - f_1)$ and $f_1 - k(f_2 - f_1)$ decrease systematically when k increases. Thus, this paper assumed that the levels of DPs below f_1 submitted to a quadratic distribution. According to this assumption, this paper calculated sound pressure levels for all DPs at frequencies lower than f_1, based on the psychoacoustic experiment data measured by Goldstein, Pressnitzer and Patterson [4, 10, 11]. When f_1 and f_2 are 1000 and 1100 Hz and the sound pressure level of each primary component is 50 dB SPL, the fitting data are shown in the table.

For a lowpass filtered $x^n(t)$, the functional model proposed by Fig. 1 has to find the optimum weights. This paper proposed to use a global optimization algorithm (simulated annealing and genetic algorithm) to search the optimum weights based on the fitting data shown in the table. The first step is to find the rough range of the optimum weights by genetic algorithm, which includes operations of selection, recombination, and mutation. The second step is to search the optimum weights within the rough range by simulated annealing algorithm, which includes two circulations. The outer circulation controls the decrease of temperature; the inner circulation controls several random searches under the same temperature. Within each search, the probability to accept a new value is determined by Eq. (6). When the new value is no more than the original value, the original value is replaced by the new value; otherwise, the original value is replaced by the new value with a certain probability. Then repeat the first and second steps until iterative error is stable. The iterative error is calculated by Eq. (7). $P_{m,i}$ is the power spectral density of a certain DP generated by the model; $P_{r,i}$ is the power spectral density of the corresponding DP shown in the table; K is the total number of DPs.

The fitting sound pressure levels of DPs

Frequency, Hz	Sound pressure level, dB SPL
900	35.7143
800	17.6184
700	5.9079
600	0.5828
500	0.8367
400	4.0931
300	10.1303
200	18.9483
100	30.5471

$$P = \begin{cases} 1 & f(j) \le f(i) \\ \exp\left(-\dfrac{f(j)-f(i)}{t_k}\right) & f(j) > f(i) \end{cases} \quad (6)$$

$$\text{error} = \sum_{i=1}^{K} (P_{m,i} - P_{r,i})^2. \quad (7)$$

When the maximum power N of the model is 5, the output of the model has the form of Eq. (8). The iterative error is shown in Fig. 2, in which the error is converged when the iterative number is larger than 7. When the iterative number is 50, the weights found by the global optimization algorithm are:

$a_2 = 0.00012622913345$,
$a_3 = 38110.2458861360$,
$a_4 = 9100204.47336136$,
$a_5 = 5876319662.05066$.

$$x_d(t) = x(t) + a_2x^2(t) + a_3x^3(t) + a_4x^4(t) + a_5x^5(t). \quad (8)$$

For DPs $f_1 - k(f_2 - f_1)$ and $k(f_2 - f_1)$, the sound pressure level decreases dramatically with increasing k [4]. The prominent components are $f_2 - f_1$, $2f_1 - f_2$, and $3f_1 - 2f_2$ [3, 6, 7]. Thus, when the maximum power of the model is 5, it is adequate to predict the sound pressure levels of these relatively prominent DPs.

Note that the weights increase with the increasing power n, i.e., $a_5 > a_4 > a_3 > a_2$. This is caused by the fact that the amplitude of $x(t)$ is less than 1 when the sound pressure level of each primary component in $x(t)$ is 50 dB SPL. When the power n increases, the amplitude of $x^n(t)$ decreases dramatically. Thus the weight of $x^n(t)$ has to be increased markedly to generate appropriate sound level for the corresponding DP.

3. THE EFFECT OF DPS ON PITCH PERCEPTION FOR UNRESOLVED HCTS

For unresolved HCTs, pitch perception depends on the temporal envelope information evoked by the interacted components [12, 13]. However, unresolved HCTs can reintroduce DPs, which might provide new information for pitch perception. This paper tried to study the effect of DPs on pitch perception for unresolved HCTs based on the proposed functional model. There were four kinds of stimuli. The first one was a two-tone HCT. The frequencies of the components were 1000 and 1100 Hz, with the fundamental frequency of 100 Hz. The sound level of each component was 50 dB SPL. The second stimulus was generated by passing the two-tone HCT through the functional model generating DPs. Thus the stimulus was the combination of the HCT and DPs at frequencies below f_1. The third stimulus was the two-tone HCT presented simultaneously with threshold equalizing noise (TEN) [14], the level of which was 40 dB/ERB_N at 1 kHz. ERB_N denotes the equivalent rectangular bandwidth of the auditory filter as determined using young listeners with normal hearing tested at a moderate sound level [15]. The fourth stimulus was the combination of two-tone HCT, TEN, and DPs. Excitation patterns (EPs) [16] expressing information of resolvability and temporal profiles (TPs) [17] expressing temporal information were calculated for all four stimuli.

EP is the distribution of internal excitation, evoked by a sound at different places of basilar membrane, related to the corresponding characteristic frequency. Since the basilar membrane can be analyzed into several auditory filters, EP can be defined as the output of auditory filter as a function of the center frequency of the filter. In this paper, EPs were calculated as described by Glasberg and Moore [15] with the modification described by Moore et al. [16]. In the experiment, the power spectral density (PSD) was calculated separately for each stimuli, which was then used as the input of the software for calculating EPs. For each sinusoidal component in the stimuli, it was filtered into different auditory filters, which resulted in a curve of the output of each filter as a function of the center frequency of the filter. The outputs of all the components were added together to obtain the EP for the stimuli. For each visible peak on the EP, peak-to-valley ratio (PVR) was calculated, by subtracting the average value of the upper and lower troughs from the peak value. If a peak has a PVR above about 2 dB, the corresponding harmonic is resolved [18]; otherwise, the harmonic is poorly resolved or unresolved. Thus, EPs can express information of resolvability.

TPs were calculated using the software platform described by Bleeck et al. [17]. The implementation was based on a linear gamma tone auditory filter bank, followed by half-wave rectification, logarithmic compression, and low pass filtering. The stimuli were the same as those used to calculate EPs. For each visible peak at fundamental frequency (100 Hz) on the TP, temporal peak height was calculated by subtracting the average value of the upper and lower troughs from the peak value. Generally, the height of peak represents the strength of temporal information and the salience of pitch evoked by the stimuli [19, 20].

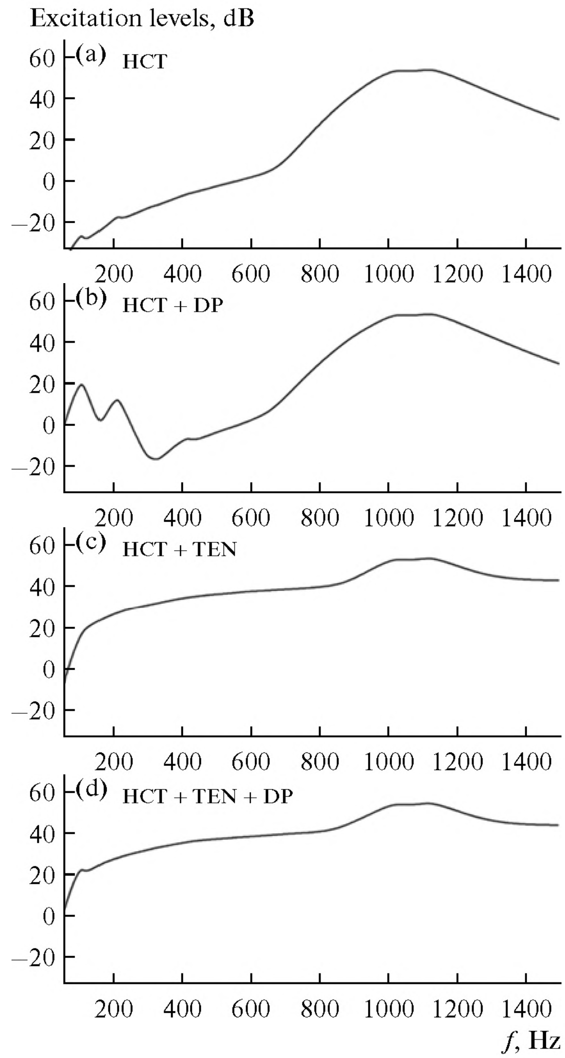

Fig. 3. Excitation patterns. Panels (a)–(d) represent excitation levels for four kinds of stimuli: (a) HCT, (b) HCT with DP, (c) HCT presented in the background noise of TEN, and (d) HCT with DP in TEN.

3.1. The Effect of DPs on EPs

The result of EPs is shown in Fig. 3. For the HCT alone (Fig. 3a), there were no visible peaks at the frequencies (1000, 1100 Hz) of primary components, which was due to the fact that the primary components were unresolved harmonics for F0 of 100 Hz and could not evoke salient peaks on the EP. When the HCT was passed through the functional model proposed in this paper, the output stimuli contained not only the primary components, but also DPs at different frequencies (Fig. 3b). The DPs at frequencies of 100 and 200 Hz not only had relatively high sound levels, but also were low-rank harmonics for F0 of 100 Hz. Thus,

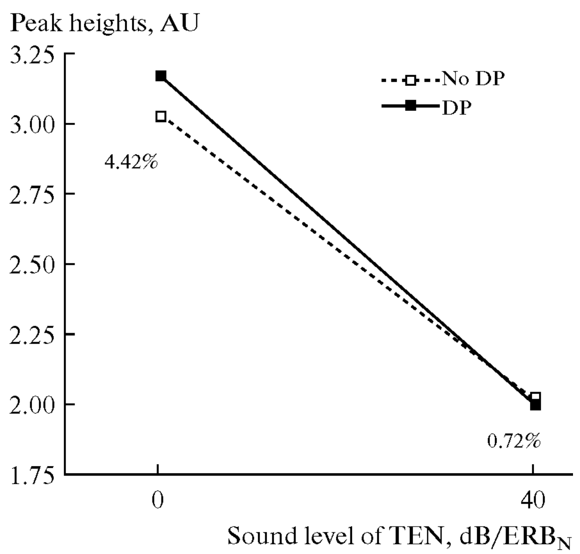

Fig. 4. Temporal peak heights. The solid line with filled squares represents cases with DPs, while the dashed line with empty squares represents cases without DPs. Stimuli can be presented in the absence of TEN (0 dB/ERB$_N$) or in the presence of TEN (40 dB/ERB$_N$).

the two DPs were resolved and could evoke salient peaks (PVRs > 9.8 dB) on the EP. These salient peaks evoked by the two DPs reflect the fact that DPs could enhance the resolvability of the HCTs. For DPs at frequencies of 900 and 800 Hz, i.e., DPs $2f_1 - f_2$ and $3f_1 - 2f_2$, although the sound levels were relatively high (as shown in table 1), they were the 9th and 8th harmonics for F0 of 100 Hz. Thus, these two components were unresolved and could not evoke salient peaks on the EP.

When TEN with the level of 40 dB/ERB$_N$ was presented with the HCT simultaneously, EPs were very similar for the cases both with and without DPs (Figs. 3c and 3d). In addition, after mixing the stimuli with TEN with the level of 40 dB/ERB$_N$, the salient peaks at frequencies of 100 and 200 Hz evoked by DPs have disappeared (Figs. 3b and 3d). Both points suggest that background noise with appropriate sound levels could mask DPs reintroduced by the harmonics, which could therefore reduce the resolvability of the unresolved HCTs and the effect of DPs on pitch perception of unresolved HCTs.

The finding that DPs could enhance the resolvability of the unresolved HCT accords with the discovery of DPs [21]. In 1714, Tartini heard a third tone when two tones sounded simultaneously [21]; this additional tone is a DP. In fact, if a component in an HCT could be heard out, this component is considered to be resolved [18]. Thus, the DPs that are heard out by human ears are resolved, which accords with the conclusion that DPs could enhance the resolvability of the HCTs.

3.2. The Effect of DPs on TPs

For TPs, the result of peak heights on TPs is shown in Fig. 4. The stimuli are the same as those in Fig. 3. The solid line with filled squares represents cases with DPs, while the dashed line with empty squares represents cases without DPs. The stimuli could be presented either in the absence of TEN (0 dB/ERB$_N$) or in the presence of TEN with the level of 40 dB/ERB$_N$.

When TEN was absent, peak height for stimuli containing DPs was 4.42% higher than those without DPs. This suggests that DPs can enhance temporal information of HCTs, which can enhance the salience of pitch evoked by the stimuli and reduce the difficulty of pitch perception. However, when TEN was presented simultaneously with the stimuli, peak height for the case with DPs was similar to that without DPs; the former was even 0.72% lower than the latter. In addition, the peak height is lower for the mixture of TEN, stimuli and DP than that for the mixture of stimuli and DP. Both points suggest that TEN with the level of 40 dB/ERB$_N$ can mask DPs, reducing the contribution of DPs to temporal information.

Since the enhancement of resolvability and temporal information of the HCTs caused by DPs are carried out simultaneously, there are nearly no papers studying the effect of DPs on resolvability and temporal information of the HCTs based on psychoacoustic experiments separately.

However, DPs indeed facilitate pitch perception of HCTs in terms of studies based on psychoacoustic experiments. For example, Moore et al. [22] measured fundamental frequency difference limens (F0DLs) for HCTs, which were presented in TEN with the level of 30 dB below that of each component in the HCTs. Since the level of TEN was very low, DPs were not completely masked, resulting in good F0DLs when the lowest harmonic rank of the HCT was in the range 8–11. In contrast, Oxenham et al. [23] replicated the experiment of Moore et al. using the same level of TEN as in the original study, and also using a level that was 20 dB higher. The TEN with the higher level masked any combination tones and resulted in worse F0DLs when the lowest harmonic rank was above 6. Considering the effect of DPs, several studies used background noise to mask potential DPs, reducing the effect of DPs on pitch perception of HCTs [22–25].

In conclusion, unresolved HCTs could reintroduce DPs with relatively high sound levels, which could enhance the resolvability and temporal information of the stimuli. If DPs are masked by background noise with appropriate sound levels, the effect of DPs is reduced and the perception of pitch for unresolved HCTs under such case is determined by the primary components of the HCTs.

4. LIMITATIONS OF THE MODEL

The proposed model has three limitations. First, the model didn't consider the effect of frequency interval on the sound levels of odd-order DPs. Previous studies found that the level of the $2f_1 - f_2$ DP decreased when the frequency interval between the primary stimuli increased [3, 4, 8, 9]. However, there have been no similar psychoacoustic findings for other DPs. The second limitation is that the current model is applied for two-tone stimuli instead of multi-tone stimuli. In order to generalize this model to more complex stimuli, more psychoacoustic data have to be collected, e.g., the effects of harmonic number, frequency interval, sound pressure level, harmonic rank on DPs at different frequencies. A more appropriate model generating DPs could be established only when these experimental data are collected. Third, this model is only aimed to generate DPs at the cochlear level, which cannot explain problems related to auditory perception at higher levels, e.g., the effect of missing fundamentals and learning effect. Future work may consider more complicated factors to build a more appropriate model.

5. CONCLUSIONS

This paper proposed a functional model generating DPs at frequencies below the primary stimuli. The main contents are as follows:

1—Summarized the mathematical relationship between DPs and odd- and even-order power of the stimuli.

2—Proposed a functional model generating DPs.

3—Used simulate annealing and genetic algorithm to search the globally optimum weights.

4—Studied the effect of DPs on the perception of pitch for unresolved HCTs based on the proposed functional model, which found that DPs could enhance the resolvability and temporal information of the HCTs. Thus the paper suggests that background noise with appropriate sound levels should be used to mask DPs to reduce the effect on pitch perception. More future work can focus on the verification based on psychoacoustic experiments.

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