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Underwater Communication Using Weakly Dispersive Modal Pulses[1]

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Abstract—In the modal description of sound propagation, signal distortion is caused primarily by dispersion, which is largely controlled by the product $I(m;f)\beta(m;f)$. Here m is mode number, f is acoustic frequency, I is the action and β is the waveguide invariant. A modal pulse with fixed m and variable f that satisfies $I\beta \approx 0$ over the entire frequency band is referred to in this letter as a weakly dispersive modal pulse. The manner by which weakly dispersive modal pulses can be exploited in underwater communications applications is described and illustrated. The connection between weakly dispersive modal pulses and weakly divergent beams is discussed.

Keywords: modal description of sound propagation, underwater communication

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Acoustic transmission of information in the ocean over horizontal distances greater than the ocean depth is challenging because propagation effects distort the transmitted signal. The usual method of overcoming signal distortion is to perform some form of channel equalization, which requires knowledge of the impulse response function of the ocean sound channel. In the ray and mode descriptions of sound propagation, signal distortion is associated with multipathing and dispersion, respectively. In this letter attention is focused on the modal description. Modal dispersion is largely controlled by the product $I(m; f)\beta(m; f)$. Here m is mode number, f is acoustic frequency, I is the action and β is the waveguide invariant. In this letter a collection of modes with fixed m and variable f (typically $f_0/\Delta f \approx 4$) that satisfy $I\beta \approx 0$ is referred to as a weakly dispersive modal pulse. It is shown, using a combination of theoretical arguments and deep water simulations, that weakly dispersive modal pulses can be exploited to transmit information in a sound channel without performing channel equalization. The connection between weakly dispersive modal pulses and weakly divergent beams is briefly discussed.

We focus in this letter on communications applications of mode-processed transient wavefields (see also [1]). Specifically, we consider broadband distributions of energy with fixed mode number. Distributions of this type are referred to in this letter as modal pulses. Note that nonzero bandwidth is essential in communication applications as the information content in any sensibly coded signal increases with increasing bandwidth. Much of the discussion in this letter is motivated by consideration of transmission of a communications signal consisting of a phase-modulated binary sequence in which each bit consists of a small number n_c of cycles of some carrier frequency; note that $f_0/\Delta f \approx n_c$. Such a sequence of bits undergoes negligible propagation-induced distortion if and only if each bit undergoes negligible propagation-induced distortion. For this reason we focus throughout this letter on the propagation-induced distortion of a single bit. After performing the requisite mode-processing, the resulting wavefield can be thought of and described as a dispersive wavetrain that, in addition, is subject to scattering effects.

We begin by illustrating the important role of the waveguide invariant β in controlling the dispersion-induced distortion as a function of range of such a dispersive wavetrain. Consider sound propagation in a stratified environment whose density is constant; $c = c(z)$, where c is sound speed and z is depth. Following [2] we define the waveguide invariant as

$$\beta(m; f) = -\frac{\partial S_g}{\partial S_p}. \qquad (1)$$

Here $S_g(m; f) = dk/d\omega$ is the group slowness where k is the radial wavenumber, and $\omega = 2\pi f$ is the acoustic wave radian frequency; $S_p = k/\omega$ is the phase slowness. The waveguide invariant was originally introduced [3, 4] as the inverse of β defined in Eq. (1). Both definitions of β are now commonly used. For our purposes Eq. (1) is most convenient. Also, in [4] β is defined using differences in S_g and S_p between neighboring mode numbers. The differential form of the definition

[1] The article is published in the original.

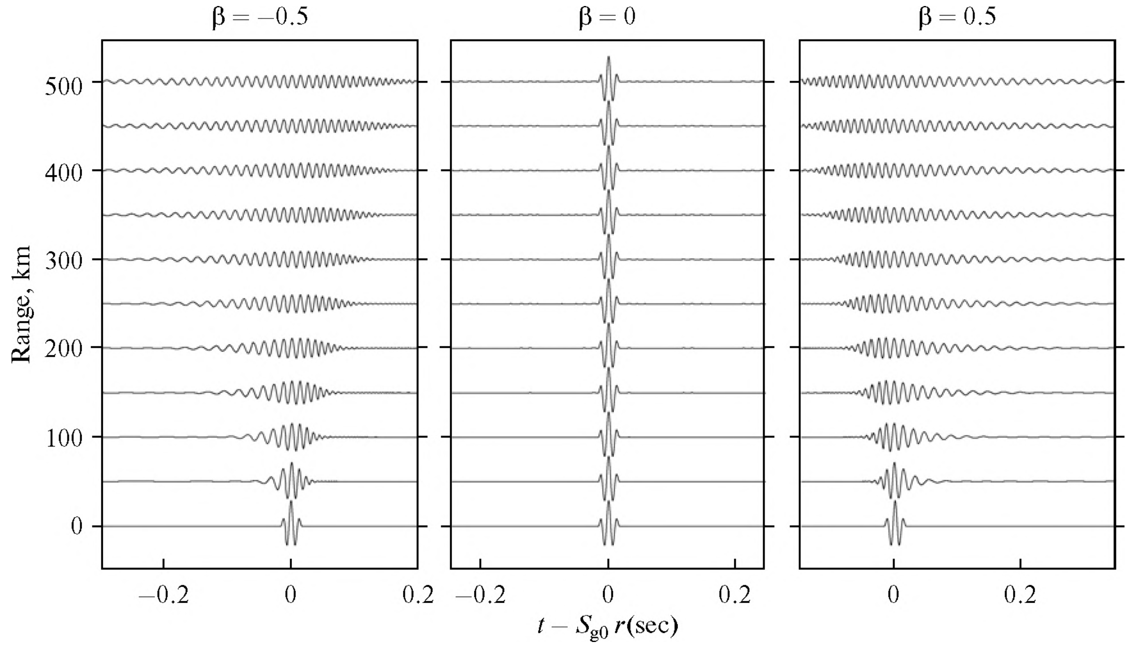

Fig. 1. Evolution in range r of idealized single-bit dispersive wavetrains with $\beta = -0.5$ (left), $\beta = 0$ (center), and $\beta = 0.5$ (right). In all three cases the single-bit wavetrain at $r = 0$ corresponds to three cycles of a 75 Hz carrier under a tapered envelope. Other parameters that were used in the simulations shown are described in the text.

of β can be thought of as a high frequency asymptotic approximation to the difference-based definition. Our use of an asymptotically valid definition of β is consistent with the asymptotic analysis that is used throughout this paper.

Assume, for now, that m is fixed and that β is constant over the frequency band of interest, centered on $f_0 = \omega_0/(2\pi)$. Then, integrating Eq. (1) gives $dk/d\omega = S_{g0} + \beta(k_0/\omega_0 - k/\omega)$, where $k_0 = k(\omega_0)$ and $S_{g0} = (dk/d\omega)(\omega_0)$. A second integration yields

$$k(\omega) = k_0\left(\frac{\omega_0}{\omega}\right)^{\beta} + \frac{S_{g0} + \beta k_0/\omega_0}{\beta + 1}\omega\left[1 - \left(\frac{\omega_0}{\omega}\right)^{\beta + 1}\right], \quad (2)$$

for $\beta \neq -1$, and

$$k(\omega) = k_0\left(\frac{\omega}{\omega_0}\right) + \left(S_{g0} - \frac{k_0}{\omega_0}\right)\omega \ln\left(\frac{\omega}{\omega_0}\right), \quad (3)$$

for $\beta = -1$. For the special case $\beta = 0$, $k(\omega) = k_0 + S_{g0}(\omega - \omega_0)$ and $S_g = dk/d\omega = S_{g0}$.

Figure 1 shows the evolution in range of three dispersive wavetrains corresponding to $\beta = -0.5$, $\beta = 0$, and $\beta = 0.5$. These wavetrains are defined by the Fourier integral

$$p_m(r, t) = \int A(\omega) e^{i\varphi(r)} e^{i(k(\omega) r - \omega t)} d\omega. \quad (4)$$

In this expression the subscript m in $p_m(r, t)$ is included to emphasize that mode number is assumed to be fixed. (One could also include a subscript m on $k(\omega)$ in Eqs. (2–4), i.e., write $k_m(\omega)$ to account for the dependence of β on m. To keep the notation simple, we have not done so). In the simulations shown in Fig. 1, $A(\omega)$ was chosen so that $\int A(\omega) e^{-i\omega t} d\omega$ is equal to three cycles of $\cos(\omega_0 t)$, centered at $t = 0$, under a cosine squared envelope with zeros at the pulse start and end times. Here $\omega_0 = 2\pi(75 \text{ Hz})$; other parameters used in the simulations were $k_0/\omega_0 = S_{p0} = 0.665$ s/km and $S_{g0} = 0.667$ s/km. The phase correction $\varphi(r) = \omega_0(S_{g0} - S_{p0})r$ is required to prevent loss of carrier frequency phase information when $\beta = 0$ and the reference group and phase slownesses are unequal. (Without the factor $e^{i\varphi(r)}$ there is no dispersion-induced pulse broadening when $\beta = 0$, but the phase under the envelope slowly drifts with increasing r if $S_{g0} \neq S_{p0}$. As a result, the information encoded in the phase of the carrier will be lost if the phase correction factor is not included.) When evaluating the integral (4) at each r, the entire integrand at each negative frequency was set equal to the complex conjugate of the integrand at the corresponding positive frequency. Figure 1 shows that when $\beta = \pm 0.5$ the single-bit wavetrain undergoes significant dispersion-induced broadening that increases with increasing range, but when $\beta = 0$ there is no dispersion-induced broadening. When β is not close to zero, neighboring bits will bleed into each other, resulting in loss of phase information encoded in each bit. It is clear from these observations that conditions are favorable for underwater communications applications using modal pulses only when β is near zero.

The foregoing discussion illustrates the importance of dispersion-induced degradation of a modal pulse and the role of β in quantifying that effect. We consider now a more quantitative description of propagation-induced degradation of modal pulses, including both dispersion and scattering effects. References [5–7] considered the temporal spreading of modal pulses in environments consisting of a range-independent background sound speed profile on which a range– and depth-dependent perturbation, due, for example, to internal waves, is superimposed. It was shown that there are three contributions to modal pulse time spreads, and that these combine approximately in quadrature, $\Delta t_m = \sqrt{\Delta t_{bw}^2 + \Delta t_d^2 + \Delta t_s^2}$. Here Δt_m is the total temporal spread of the modal pulse, Δt_{bw} is the reciprocal bandwidth of the modal pulse, Δt_d is the dispersive contribution to the pulse spread, and Δt_s is the scattering contribution to the pulse spread. At $r = 0$ both Δt_d and Δt_s vanish, so $\Delta t_m(r = 0) = \Delta t_{bw}$; Δt_{bw} should thus be thought as the minimal, bandwidth-dependent, width of the modal pulse. Δt_{bw} is the duration of a single bit of the transmitted communications signal; that term will not be further discussed. In Ref. [6] it is shown that $\Delta t_{bw} = 1/\Delta f$,

$$\Delta t_d = 2\pi \frac{\Delta f}{f_0} \frac{|\beta(I)|}{R(I)} I r, \tag{5}$$

and

$$\Delta t_s = 4\pi^{3/2} \left(\frac{B}{3}\right)^{1/2} \frac{|\beta(I)|}{R(I)} r^{3/2}. \tag{6}$$

Here I is the classical action (see, e.g., [8]), and $R(I)$ is the range double loop distance of a ray or mode whose action is I. A quantization condition relates the action to the acoustic frequency and mode number. Asymptotically, for modes with two internal turning points, appropriate for describing usual deep ocean conditions, the quantization condition is $\omega I = m + 1/2$, $m = 0, 1, 2, \ldots$. The quantization condition allows dependence on I to be traded for dependence on frequency and mode number in the above expressions. Variations in $R(I)$ in typical deep ocean environments are quite small. With this in mind, consistent with Eq. (5), we define a weakly dispersive group of modes as one for which $\beta I \approx 0$.

A direct argument leading to the conclusion that dispersive spreading is controlled by βI follows from consideration of the decompression coefficient, $\gamma = \partial^2 k/d\omega^2$ (partial derivatives are evaluated here at fixed mode number). The phase slowness $S_p = k/\omega$ is quantized, $S_p = S_{pm}$, via the quantization condition with $I(S_{pm}) = (2\pi)^{-1}\oint (c^{-2}(z) - S_{pm}^2)^{1/2} dz$. Note that $S_g = \partial k/d\omega$ and $\beta = -\partial S_g/\partial S_p$ gives $\gamma = -\beta/(\partial\omega/\partial S_{pm})$. Finally, $\partial\omega/\partial S_{pm} = (\partial\omega/\partial I)(dI/dS_{pm}) = (-\omega/\mathrm{I})(-R(I)/(2\pi))$, so $\gamma = -\beta I/(fR(I))$. The same combination of terms appears in Eq. (5).

Consider now the scattering-induced modal pulse spreading term, Δt_s, described by Eq. (6). The quantity B in that expression is the action diffusivity; owing to scattering by small-scale ocean structure, mean-square spreads in action grow like Br. The arguments leading to Eq. (6) require modification for near-axial (small I or low m) modes in deep ocean environments; those modes are scattered less strongly than is suggested by Eq. (6). Unfortunately, the extended theory [7–10] does not lead to a simple analytical expression for Δt_s. The dependence on $|\beta(I)|$ in Eq. (6) carries over to the extended theory, however. With these comments in mind, it is seen that the scattering-induced time spread for a weakly dispersive modal pulse, for which $\beta I \approx 0$, is expected to be small.

In practice there are two classes of modes that satisfy our definition of a weakly dispersive modal pulse. First is the $m = 0$ (near-axial) modal pulse. That modal pulse has very small I (equal to $(2\omega)^{-1}$ at each frequency). Additionally, in any smooth single-minimum sound speed profile $\beta \longrightarrow 0$ as $I \longrightarrow 0$, so the smallness of I for these modes implies smallness of β. Because βI is small for this modal pulse Δt_d is small by Eq. (5). The $m = 0$ modal pulse exhibits small scattering-induced time spreads for several reasons: 1—the smallness of β (recall Eq. (6)); 2—the near-axial correction to Eq. (6) that we mentioned; and 3—because near-axial sound speed fluctuations are generally much weaker than upper ocean sound speed fluctuations. (Dependence on scattering strength in Eq. (6) enters through the action diffusivity B). The second class of modes that satisfy our weakly dispersive definition are modes with nonzero m for which $\beta(m; f_0) \approx 0$. Equations (5) and (6) reveal that such modes exhibit both small dispersive spreading and small scattering-induced spreading. Note that because $\beta(m; f_0)$ is a discretely sampled function, it is generally not possible to satisfy the condition $\beta(m; f_0) = 0$. Typically, for the m that is chosen, there are some frequencies in the excited band for which $\beta < 0$, one frequency—usually not the carrier—for which $\beta = 0$, and some frequencies for which $\beta > 0$. It is not possible in general to satisfy the condition $\beta = 0$ over the entire excited frequency band; the best that one can do is to include a zero-crossing in the excited frequency band and require small $|\beta|$ over the entire band. This requirement becomes increasingly difficult to satisfy as the bandwidth increases. With these comments in mind, it should be clear that the $\beta = 0$ simulation shown in Fig. 1 is highly idealized.

Acoustic simulations based on the RAM propagation model [11] were performed to test the ideas that we have described; the results are presented in Fig. 2. The environment used in the simulations consisted of a range-independent deep ocean sound speed profile

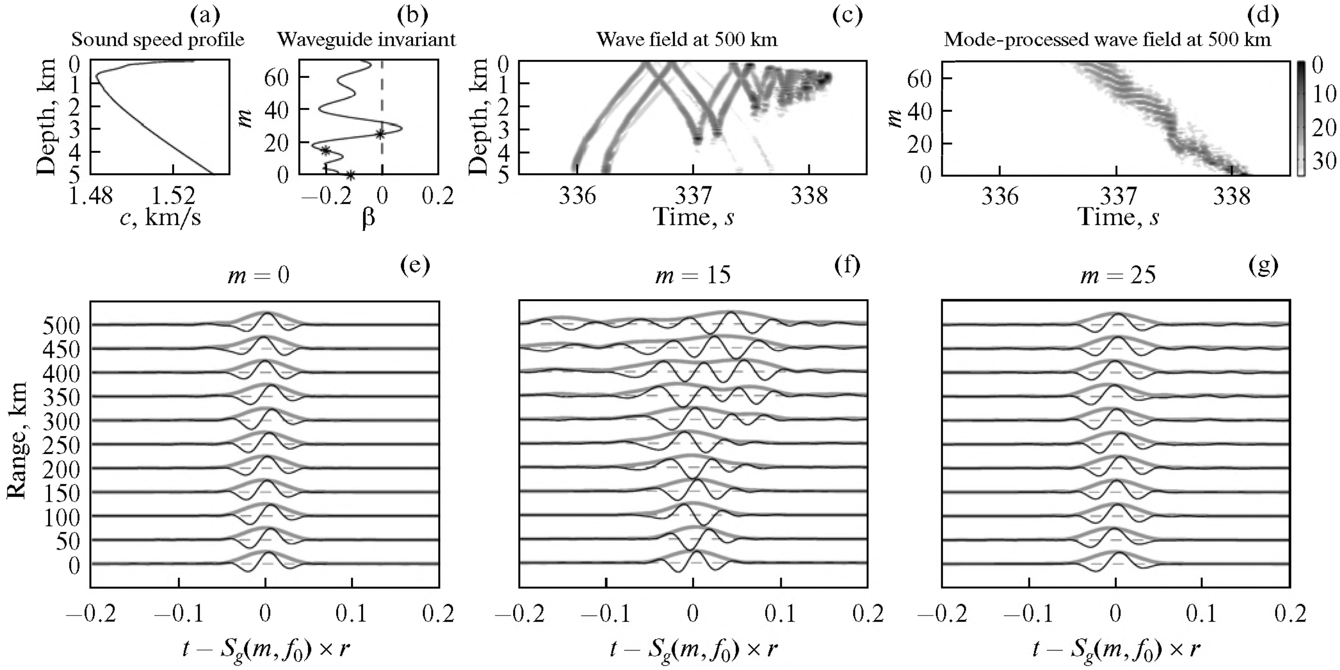

Fig. 2. a—Background sound speed profile used in wavefield simulation. b—Corresponding β vs m curve at $f_0 = 75$ Hz. Mode numbers 0, 15 and 25 are marked with an asterisk. c—Simulated wavefield $p(z,t)$ at a range of 500 km for a source at 800 m depth with $f_0 = 75$ Hz and a computational bandwidth of 37.5 Hz. d—Corresponding mode-processed wavefield $p_m(t)$. Wavefield intensity, with a 35 dB dynamic range, is plotted in both (c) and (d). The range evolution of the $m = 0$, 15 and 25 modal pulses are shown in (e), (f) and (g), respectively. The envelope of each pressure time history is shown with a heavy gray curve.

typical of early Fall conditions in the eastern North Pacific Ocean (Fig. 2a), on which an internal-wave-induced sound speed perturbation [12] was superimposed. The near-axial source transmitted a pulse consisting of four cycles of a 75 Hz carrier (the effective bandwidth was approximately half of the 37.5 Hz computational bandwidth). The resulting wavefield $p(z, t)$ (Fig. 2c) and corresponding mode-processed wavefield $p_m(t)$ (Fig. 2d) are shown at a range of 500 km. Note that the multipath time spread is about 2 sec and that the time spread of most modal pulses is several tenths of a sec, while the duration of the transmitted pulse (one bit) is about 50 ms. Also shown in the figure are $\beta(m; 75\text{ Hz})$ (Fig. 2b) and the range evolution of the modal pulses corresponding to $m = 0$ (Fig. 2e), $m = 15$ (Fig. 2f) and $m = 25$ (Fig. 2g). $\beta(m;$ 75 Hz$)$ has zeros near $m = 25$ and $m = 32$. The $m = 0$ and $m = 25$ modal pulses correspond, according to our definition, to weakly dispersive modal pulses. The $m =$ 15 modal pulse is not a weakly dispersive modal pulse, according to our definition; that modal pulse is included in the figure to illustrate the difference between weakly dispersive modal pulses and more typical modal pulses. In the construction of the $m = 0$ and $m = 25$ modal pulses shown in Fig. 2, phases were adjusted using the phase factor $e^{i\varphi(r)}$ that was discussed in the context of Eq. (4). This phase correction will be discussed in more detail below.

Figure 2 shows that at 500 km range both the $m = 0$ and $m = 25$ modal pulses have experienced very little dispersion-induced distortion, while the $m = 15$ modal pulse has experienced very significant dispersion-induced distortion. It follows that if a sequence of bits, each consisting of a few cycles of a 75 Hz carrier with some appropriate phase-modulation, is transmitted, that same sequence of bits can be recovered at $r =$ 500 km by constructing the corresponding $m = 0$ and $m = 25$ modal pulses. In other words, the $m = 0$ and $m = 25$ modal pulses provide a basis for transmitting a communications signal at $r = 500$ km in this environment without performing any channel equalization. The same cannot be said of almost all other modal pulses, however, including the $m = 15$ modal pulse. We have not considered ranges longer than 500 km in our simulations. Beyond that range the assumption of a range-independent background sound speed profile will, in most ocean environments, be questionable. Also, keep in mind that our simulations don't account for ambient noise, measurement errors, source and/or hydrophone positioning errors, source motion, etc.

Two subtle issues relating to the simulations shown in Figs. 2e–2g deserve further discussion. First, note that the $m = 0$, 15 and 25 modal pulses at very short range are not identical to each other and all differ slightly from the source time history. The reason for this is that, across the excited frequency band, the excitation of modes with those mode numbers is not uniform. In practice, to insure that the m-th modal pulse at very short range is a good approximation to the source time history, the source depth must approximately coincide with an antinode of a modal eigenfunction $\psi_m(z; f_0)$. Second, our simulations revealed that, in the presence of an internal-wave-induced sound speed perturbation, estimates of S_{g0} and S_{p0} based on the background sound speed profile were not accurate enough for the phase correction factor $e^{i\varphi(r)}$ to freeze the phase of the carrier frequency under the source envelope for $m = 0$ and $m = 25$. To solve this problem $S_{g0} - S_{p0}$ was chosen to be the smallest number that shifts the phase of the carrier in the modal pulse at $r = 500$ km in such a way as to coincide with the phase of the carrier of the source function; the same (linearly increasing with range) phase correction was then applied at all ranges shown in Figs. 2e and 2g. Experimentally, this procedure could be duplicated by transmitting one or more phase calibration bits in addition to the communications signal. Note also that this phase correction procedure is convenient but not essential; if this correction is not applied phase *differences* between 0-bits and 1-bits will be maintained, thereby allowing the communications signal to be recovered. No phase correction was applied to the $m = 15$ modal pulse shown in Fig. 2f because the idea of freezing the phase under the envelope is meaningful only if the envelope has a fixed form, i.e., only for weakly dispersive modal pulses.

Recall that we defined weakly dispersive modes as those for which $I\beta(I) \approx 0$. The definition $I\beta(I) \approx 0$ was used rather than $\beta(I) \approx 0$ so as to include the $m = 0$ mode. If we neglect the $m = 0$ mode and invoke the slightly more restrictive definition that a mode is weakly dispersive if $\beta(I) = 0$, then one sees that weakly dispersive modes are closely related to weakly divergent rays [13–19]. This connection follows from the asymptotic equivalence [20] of the ray-based stability parameter $\alpha(I)$ and the mode-based waveguide invariant $\beta(I)$, and the observation that weakly divergent rays satisfy the condition $\alpha(I) = 0$. (Note that the action I can be used to label both rays and modes). $\alpha(I)$ is defined as $(I/\Omega)d\Omega/dI$ where $\Omega(I) = 2\pi/R(I)$ is the spatial 'frequency' of a ray. Weakly divergent rays satisfy the condition $dR/dS_p = 0$, or, equivalently, $\alpha = 0$. (It is helpful here to note that the phase slowness S_p is also the horizontal component of the ray slowness vector, which is constant following a ray in a stratified environment, and that $-2\pi dI/dS_p = R$.) A weakly divergent beam is a continuum of rays surrounding a ray with $\alpha = 0$, while a weakly dispersive modal pulse is a continuum of modes surrounding a mode with $\beta = 0$. Thus, apart from the fact that ray-mode equivalence dictates that a mode is formed by interfering up– and down-going rays whose turning depths coincide with the modal turning depths, weakly divergent beams and weakly dispersive modal pulses are essentially the same distributions of energy. Both weakly dispersive modal pulses and weakly divergent beams are known to have many interesting properties; in addition to the references listed above, see [21] and references therein.

In this letter we have pointed out that weakly dispersive modal pulses, consisting of distributions of acoustic energy with fixed mode number and variable frequency, for which $I(m; f)\beta(m; f) \approx 0$ over the entire frequency band, are well suited for underwater communications applications. Such weakly dispersive modal pulses are advantageous because they are associated with both small dispersion-induced distortion and small scattering-induced distortion. By exploiting weakly dispersive modal pulses, communication signals can be transmitted from a point source to a distant location without performing channel equalization. A vertical line receiving array is required because mode filtering must be performed to extract the transient signal carried by certain fixed mode numbers. In practice, the need to measure the wavefield on a vertical line array should be largely offset by a significant processing gain associated with the required mode-processing. Although the procedure that we have described avoids channel equalization, it should be kept in mind that to perform the processing that we have described, the environment must be known well enough to compute $\beta(m; f_0)$ and to construct $\psi_m(z; f)$, across the relevant frequency band, for one or more weakly dispersive modes. The close connection between weakly dispersive modal pulses and weakly divergent beams has been pointed out.

Finally, we note that the procedure that we have described has been implemented using data collected (for a different purpose) in a deep ocean environment with $f_0 = 75$ Hz and $n_c = 2$. The first ten modes were used in that analysis because the receiving array deployed in that experiment limited analysis to low mode numbers. Using those data, the procedure that we have described works very reliably at 50 and 250 km, and somewhat reliably at 500 km. Experimental results will be described elsewhere.

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