CLASSICAL PROBLEMS OF LINEAR ACOUSTICS AND WAVE THEORY

Electroseismic Waves Excited by Vertical Magnetic Dipole in Borenole[1]

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Abstract—Acoustic and electromagnetic fields are coupled in a fluid saturated porous medium due to seismoelectric effect. Seismoelectric well logging method has been proposed to detect deep target formation utilizing such effect. Because of uncoupling of SH waves with P-SV waves, a simple and forthright way to get shear waves information is possible, especially for soft or slow formation whose shear wave velocity is lower than the velocity of borehole fluid. We consider the wave fields excited by a vertical magnetic dipole (VMD) source. Two methods are used to simulate, one is the coupled method based on Pride model and the other is the uncoupled method. For two methods, the frequency wavenumber domain representations of the acoustic field and associated seismoelectric field are formulated. The full waveforms of acoustic waves and electromagnetic wave induced SH waves excited by VMD source in the time domain propagation in borehole are simulated and analyzed.

Keywords: porous medium, seismoelectric effect, vertical magnetic dipole, SH waves.

INTRODUCTION

The investigation of wave propagation in fluid-saturated porous media was early developed by Biot [1, 2]. Wave propagation in fluid-saturated porous media always captures the interest of many scientists in acoustics due to its important applications in various technical and engineering processes (see, e.g. [3–8] and literature cited there). The practical interest in connection with the geophysics, acoustic wave in cylindrical structure is given more attention on [9–15], especial in a borehole embedded in fluid-saturated porous formation [16–21].

In a fluid saturated porous formation, at a solid grain-fluid interface of a porous medium where a fluid electrolyte comes into contact with grain surface, anions from the fluid electrolyte are chemically adsorbed to the surface leaving behind a net excess of cations distributed near the wall. This region is known as the electric double layer (EDL). In such porous medium, acoustic and electromagnetic fields are coupled due to seismoelectric effect which related to the EDL [3, 4]. Seismoelectric effect is also called acoustoelectric effect in acoustical logging frequencies (kHz). Acoustoelectric effect well logging method has been proposed to detect deep target formation utilizing such effect [17, 18].

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Because of uncoupling of SH waves with P-SV waves, a simple and forthright way to get shear waves information is possible [10, 12], especially for soft or slow formation whose shear wave velocity is lower than the velocity of borehole fluid. It might be possible for SH-TE seismoelectric log in logging while drilling case due to running drill collar. The property of SH-TE seismoelectric wave fields in a fluid filled borehole surround by a porous media is worth to be considered. The seismoelectric wave fields excited by shear source are considered in [19], and in the presented paper we will consider the electroseismic waves excited by a VMD source in borehole. Two methods are used to simulate, one is the coupled method based on Pride model and the other is the uncoupled method [18]. For two methods, the frequency wavenumber domain representations of electroseismic field are formulated. The full waveforms of electromagnetic wave induced SH waves excited by VMD source in the time domain propagation in borehole are simulated and analyzed.

FORMULATIONS

For the problem of modeling the propagation of coupled electromagnetic and mechanical disturbances in an isotropic-porous material, Pride (1994) [3] has derived equations that control such "seismoelectric" phenomena. According to Pride's theory, no applied

force and electric current sources, time dependence $e^{-i\omega t}$, we may written the equations for the coupled electromagnetic and acoustics in macroscopically homogenous, isotropic, fluid saturated porous media as follows

$$\nabla \cdot \boldsymbol{\tau} = -\omega^2(\rho \mathbf{u} + \rho_f \mathbf{w}), \tag{1}$$

$$\boldsymbol{\tau} = (H - 2G_b)(\nabla \cdot \mathbf{u})\mathbf{I} + C(\nabla \cdot \mathbf{w})\mathbf{I} \tag{2}$$

$$+ G_b(\nabla \mathbf{u} + \nabla \mathbf{u}^T),$$

$$-p_f = C(\nabla \cdot \mathbf{u}) + M(\nabla \cdot \mathbf{w}), \tag{3}$$

$$-i\omega \mathbf{w} = (-\nabla p_f + \rho_f \omega^2 \mathbf{u})\kappa(\omega)/\eta + L(\omega)\mathbf{E}, \tag{4}$$

$$\mathbf{J} = \sigma(\omega)\mathbf{E} + (-\nabla p + \rho_f \omega^2 \mathbf{u})L(\omega), \tag{5}$$

$$\nabla \times \mathbf{E} = i\omega \mu \mathbf{H}, \tag{6}$$

$$\nabla \times \mathbf{H} = -i\omega \varepsilon \mathbf{E} + \mathbf{J}, \tag{7}$$

$$\mathbf{B} = \mu \mathbf{H}, \tag{8}$$

$$\mathbf{D} = \varepsilon \mathbf{E}. \tag{9}$$

Those governing equations are the Biot equations for porous media acoustic along with the Maxwell equations for the electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$. Here $\boldsymbol{\tau}$ is the bulk stress in the porous medium, p_f is the pressure in the pore fluid, $\mathbf{u}$ is the displacement in the solid, and $\mathbf{w}$ is the relative fluid-solid motion. $\mathbf{I}$ is an unit vector. The symbol ρ denotes the bulk density of the porous medium, $\rho = (1 - \phi)\rho_s + \phi\rho_f$, ϕ is the porosity of the medium, ρ_s is the solid density and ρ_f denotes the fluid density. The Eqs. (4) and (5) are in the form of Darcy law and Ohm law, through which acoustic and electromagnetic fields are coupled, where $\mathbf{J}$ is the electric-current density and $-i\omega \mathbf{w}$ is the Darcy filtration velocity. Where H, C, M, and G_b are four modules of isotropic porous media [22]. As the most important coefficient $L(\omega)$ is set to zero, Pride's equations will be separated into Biot's equations for elastic field and Maxwell equations for electromagnetic field. Here $\sigma(\omega)$ is the frequency-dependent electrical conductivity of the medium, $\kappa(\omega)$ is the dynamic permeability, η is the fluid dynamic viscosity, and $L(\omega)$ is the frequency-dependent electrokinetic coupling coefficient. The expressions for $\sigma(\omega)$, $\kappa(\omega)$ and $L(\omega)$ are given in Pride et al. (1994).

Here we will consider a vertical magnetic dipole source is located in the borehole axis, and there are neither acoustic nor EM source in the porous formation. A magnetic dipole can be realized with the model of a small current loop with radius a_T and $m_0 = i_0 \pi a_T^2$ is the magnetic moment of the loop where i_0 is the current in the loop. The electric fields of a magnetic dipole in a borehole can be written as

$$E_\theta = -\frac{i\theta \mu m_0}{4\pi} \frac{\partial}{\partial r}\left(\frac{\exp(ik_e R)}{R}\right). \tag{10}$$

According to the theory of Bessel functions, the following equation exists

$$\frac{\exp(ik_e R)}{R} = \frac{1}{\pi} \int_{-\infty}^{\infty} K_0(\eta_e r)e^{ik_z Z}(dk_z), \tag{11}$$

where $\eta_e^2 = k_z^2 - k_e^2$ is the radial wavenumber of the electromagnetic waves in borehole fluid. Therefore, in the frequency wavenumber domain the electromagnetic fields in the borehole can be written as

$$E_\theta = A\frac{i\omega \mu}{\eta_e}I_1(\eta_e r) + \frac{i\omega \mu m_0}{4\pi^2}\eta_e K_1(\eta_e r), \tag{12}$$

$$H_z = AI_0(\eta_e r) - \frac{\eta_e^2 m_0}{4\pi^2}K_0(\eta_e r), \tag{13}$$

$$H_r = A\frac{-ik_z}{\eta_e}I_1(\eta_e r) - \frac{ik_z \eta_e m_0}{4\pi^2}K_1(\eta_e r). \tag{14}$$

In the porous formation the acoustic field induced by EM wave and the converted acoustic field influences back on the EM field due to seismoelectric effect. The TE wave in the porous formation is coupled with SH wave due to seismoelectric effect. The solution to the coupled equations of seismoelectric waves' motions can be obtained by separating the basic field $\mathbf{u}$, $\mathbf{w}$, $\mathbf{E}$ into its horizontally polarized shear components, and can be written in terms of scalar potential functions as

$$\mathbf{u} = \nabla \times (\Psi_{sh} + \Psi_{em})\mathbf{e}_z, \tag{15}$$

$$\mathbf{w} = \nabla \times (\alpha_{sh}\Psi_{sh} + \alpha_{em}\Psi_{em})\mathbf{e}_z, \tag{16}$$

$$\mathbf{E} = \nabla \times (\beta_{sh}\Psi_{sh} + \beta_{em}\Psi_{em})\mathbf{e}_z. \tag{17}$$

Each of potentials satisfies Helmholtz-type wave equation. The expressions for α_j, β_j are given in [4], and $j = $ sh, em denotes SH waves and TE waves, respectively. The potential in the frequency-axial wavenumber is

$$\Psi_j(r, k_z, \omega) = A_j K_0(\eta_j, r), \tag{18}$$

where $\eta_j^2 = k_z^2 - k_j^2$ ($k_j^2 = \omega^2 s_j^2$) are the radial wavenumber of the seismoelectric waves and s_j is the wave slowness. The interest quantities could be expressed by the potentials.

$$u_\theta = A_{sh}\eta_{sh}K_1(\eta_{sh}r) + A_{em}\eta_{em}K_1(\eta_{em}r), \tag{19}$$

$$E_\theta = A_{sh}\beta_{sh}\eta_{sh}K_1(\eta_{sh}r) + A_{em}\beta_{em}\eta_{em}K_1(\eta_{em}r), \tag{20}$$

$$H_z = iA_{sh}\eta_{sh}^2\beta_{sh}K_0(\eta_{sh}r)$$

$$+ A_{em}\eta_{em}^2\beta_{em}K_0(\eta_{em}r)]/(\mu\omega), \tag{21}$$

$$\tau_{r\theta} = G_b[A_{sh}\eta_{sh}^2 K_2(\eta_{sh}r) + A_{em}\eta_{em}^2 K_2(\eta_{em}r)]. \tag{22}$$

We can solve the coupled governing equations and use waves boundary conditions to obtain solutions of TE-SH electroacoustic logging. Three boundary conditions at the borehole wall are

$$\tau_{r\theta} = 0, \quad E_{\theta} = E_{\theta}^{b}; \quad H_{z} = H_{z}^{b}, \qquad (23)$$

and can be rewritten in the form of matrix as,

$$\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \begin{pmatrix} A \\ A_{\mathrm{sh}} \\ A_{\mathrm{em}} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ b_{3} \end{pmatrix} \qquad (24)$$

and elements of m_{ij} are

$$m_{11} = 0; \quad m_{12} = G_{b}\eta_{\mathrm{sh}}^{2}K_{2}(\eta_{\mathrm{sh}}a);$$

$$m_{13} = G_{b}\eta_{\mathrm{em}}^{2}K_{2}(\eta_{\mathrm{em}}a); \quad m_{21} = -i\omega\mu I_{1}(\eta_{e}a)/\eta_{e};$$

$$m_{22} = \beta_{\mathrm{sh}}\eta_{\mathrm{sh}}K_{1}(\eta_{\mathrm{sh}}a); \quad m_{23} = \beta_{\mathrm{em}}\eta_{\mathrm{em}}K_{1}(\eta_{\mathrm{em}}a); \qquad (25)$$

$$m_{31} = K_{0}(\eta_{e}a); \quad m_{32} = -i\beta_{\mathrm{sh}}\eta_{\mathrm{sh}}^{2}K_{0}(\eta_{\mathrm{sh}}a)/(\omega\mu);$$

$$m_{33} = -i\beta_{\mathrm{em}}\eta_{\mathrm{em}}^{2}K_{0}(\eta_{\mathrm{em}}a)/(\omega\mu).$$

The element of b_{j} are

$$b_{1} = 0; \quad b_{2} = \frac{i\omega\mu m_{0}}{4\pi^{2}}\eta_{e}K_{1}(\eta_{e}a);$$

$$b_{3} = \frac{\eta_{e}^{2}m_{0}}{4\pi^{2}}K_{0}(\eta_{e}a). \qquad (26)$$

If the influence of the converted acoustic field on the EM field is weak enough, we can assume that it is negligible. The second term on the right hand of Eq. (5) becomes zero, thus the EM field in the porous formation can be obtained by solving Maxwell's equations. Once knowing the EM field, we can determine the converted acoustic field. The acoustic fields are satisfied

$$G_{b}\nabla^{2}\mathbf{u} + \omega^{2}\rho\mathbf{u} + \omega^{2}\rho_{f}\mathbf{w} = 0 \qquad (27)$$

$$-i\omega\mathbf{w} = \omega^{2}\rho_{f}\frac{\kappa}{\eta}\mathbf{u} + L\mathbf{E} \qquad (28)$$

i.e., the electric field can be considered as the source of acoustic field

$$G_{b}\nabla^{2}\mathbf{u} + \omega^{2}\rho\mathbf{u} - \omega^{2}\frac{\rho_{f}^{2}}{\tilde{\rho}}\mathbf{u} = -i\omega\rho_{f}L\mathbf{E}, \qquad (29)$$

here $\tilde{\rho} = i\eta/(\omega\kappa)$.

If the electric field becomes zero the above equation is Biot equation for SH acoustic waves. We can obtain the displacement induced by electric field,

$$\mathbf{u}^{*} = \ell\mathbf{E}, \qquad (30)$$

$$\ell = \frac{iL\rho_{f}}{\omega G_{b}(s_{\mathrm{em}}^{2} - s_{\mathrm{sh}}^{2})} \approx \frac{iL\rho_{f}}{\omega G_{b}s_{\mathrm{sh}}^{2}}. \qquad (31)$$

Electric field and axial magnetic field can be written as

$$E_{\theta} = -i\omega\mu B_{\mathrm{em}}K_{1}(\eta_{\mathrm{em}}r)/\eta_{\mathrm{em}}, \qquad (32)$$

$$H_{z} = B_{\mathrm{em}}K_{0}(\eta_{\mathrm{em}}r). \qquad (33)$$

With the EM waves boundary conditions at the borehole wall, i.e., $E_{\theta} = E_{\theta}^{b}$; $H_{z} = H_{z}^{b}$, the coefficient B_{em} can be determined as

$$B_{\mathrm{em}} \qquad (34)$$
$$= \frac{K_{0}(\eta_{e}a)I_{1}(\eta_{e}a) + K_{1}(\eta_{e}a)I_{0}(\eta_{e}a)}{K_{0}(\eta_{\mathrm{em}}a)I_{1}(\eta_{e}a) + \eta_{e}K_{1}(\eta_{\mathrm{em}}a)I_{0}(\eta_{e}a)/\eta_{\mathrm{em}}} \frac{m_{0}\eta_{e}^{2}}{4\pi^{2}}.$$

Displacement in porous formation can be written as

$$u_{\theta} = B_{\mathrm{sh}}K_{1}(\eta_{\mathrm{sh}}r) - i\ell\omega\mu B_{\mathrm{em}}K_{1}(\eta_{\mathrm{em}}r)/\eta_{\mathrm{em}}. \qquad (35)$$

With the acoustic field boundary condition, $\tau_{r\theta} = G_{b}(\partial u_{\theta}/\partial r - u_{\theta}/r) = 0$ or $\partial u_{\theta}/\partial r - u_{\theta}/r = 0$, the coefficient B_{sh} can be determined as

$$B_{\mathrm{sh}} = i\ell\frac{\omega\mu}{\eta_{\mathrm{sh}}}\frac{K_{2}(\eta_{\mathrm{em}}a)}{K_{2}(\eta_{\mathrm{sh}}a)}B_{\mathrm{em}}. \qquad (36)$$

It shows the conversion acoustic field and electric field is mainly dependent on the electrokinetic coupling coefficient. With the above equations, we can obtain the transient waveforms of the electric, magnetic fields and acoustic fields along the borehole by digital Fourier transform.

NUMERICAL RESULTS

The parameters of the porous formation and borehole fluid used for simulation are listed in table. A fast formation is a rock in which the shear velocity faster than the borehole fluid velocity and a slow formation is a rock in which the shear velocity slower than the borehole fluid velocity. The porous medium is a slow formation whose shear wave velocity 935 m/s, and the porous medium is a fast formation whose shear wave velocity 1707 m/s. To simulate the TE-SH electroseismic logging response, we calculate circumferential displacement u_{θ} and circumferential electric field E_{θ}. The acoustic receivers are assumed to be placed in the borehole wall and electric field sensors are placed in the fluid nearby the borehole wall. We use the pulse source function [16],

$$V_{0}(t)$$

$$= \begin{cases} \frac{1}{2}\left[1 + \cos\frac{2\pi}{T}\left(t - \frac{T}{2}\right)\right]\cos 2\pi f_{0}\left(t - \frac{T}{2}\right), 0 \leq t \leq T, \\ 0, t < 0 \text{ or } t > T, \end{cases} \qquad (37)$$

where f_{0} is the center frequency and T is the pulse width. In the following numerical simulation examples, we adopted central frequency 4 kHz and pulse width 0.75 ms.

Figure 1 shows full waveforms excited by a vertical magnetic dipole source located in the borehole surrounded by fast formation. The waveforms of responses by two different methods only have a very small relative difference, and can not be seen in figures. It is shown that if electric field is on the order of

The parameters are used for simulation. ε_0 is the permittivity of vacuum

Parameter	Property (unit)	Fast Formation	Slow Formation
a	Borehole radium, m	0.1	0.1
ϕ	Porosity, %	0.15	0.3
κ_0	Darcy permeability, Darcy	1	1
G_b	Frame shear modulus, GPa	7.0	1.855
ρ_s	Solid density, Kg/m^3	2650	2650
ρ_f	Fluid density, Kg/m^3	1000	1000
ε_s	Solid permittivity (ε_0)	4	4
ε_f, ε_b	Pore, borehole Fluid permittivity	80	80
C_f, C_b	Pore, borehole Fluid salinity, mol/L	0.001	0.001
η	Fluid viscosity, Pa · s	0.001	0.001
T	Temperature, K	298	298
α_∞	Tortuosity	3	2

1 V/m, the particle displacement is on the order of picometre. Obviously, there are two different wave packets of particle displacement. It is shown by lines a–a and b–b in Fig. 2. Two wave packets are originated at the borehole wall through conversion of EM energy to acoustic energy. The first wave packets (line a–a) reach the receivers at almost the same time which named as an EM-accompanying acoustic wave groups. Unlike vertical electric dipole source as a source in E-A logging [18], which will excited the compressional wave group, the shear and pseudo-Rayleigh wave group and the Stoneley wave group in the fast formation. Vertical magnetic dipole source will only excited the shear wave group (TE-SH waves).The difference in time between arrivals at the receivers is used to estimate the travel time, or slowness (the inverse of acoustic velocity), of acoustic waves in formation. We can calculate the second wave packets propagate with shear wave velocity about 1707 m/s.

It is well known that conventional sonic logging tool can't be used to obtain the shear wave velocity in slow formation. Modern sonic logging tool carries both monopole and dipole source and receivers so that compressional and shear waves' arrivals can be recorded in fast formation and slow formation [9, 16, 21]. The dipole sonic log can generate a flexural wave in slow formation. The flexural wave is dispersive, i.e. low frequency travel faster than high frequency. The lowest frequency component arrives at shear wave velocity. It needs for one conduct a correction (slowness) from dipole waves (flexural mode). If we use one type shear waves, SH-TE or TE-SH waves, we won't need to conduct slowness correction. Full waveforms excited by a vertical magnetic dipole source located in the borehole surrounded by slow formation are shown in Fig. 3. Again the waveforms of responses by two different methods only have a very small relative difference, and can not be seen in figures. There are two dif-
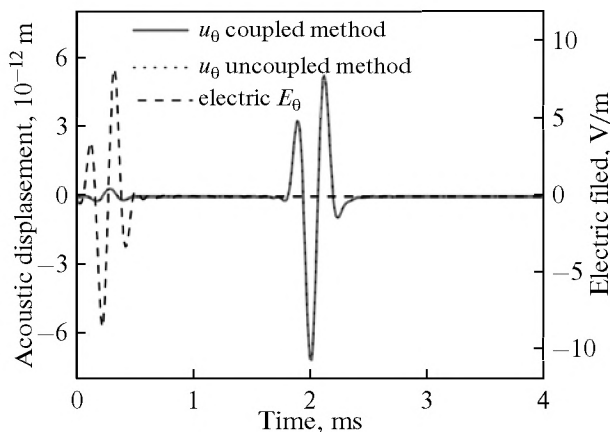

Fig. 1. Transient full waveforms of electric field near the borehole wall and acoustic displacement of the borehole wall for the fast formation.
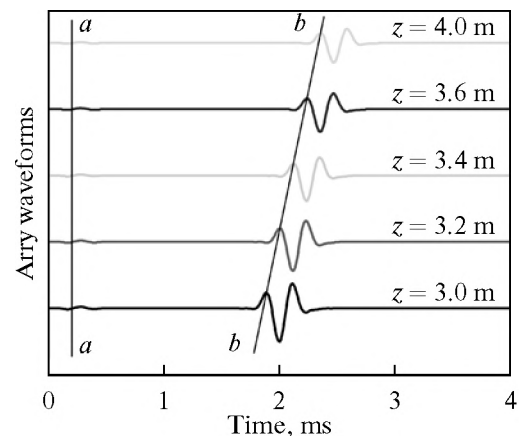

Fig. 2. Transient full waveforms of acoustic displacement of the borehole wall for the fast formation when the distance to the source z varies from 3.0 to 4.0 m.
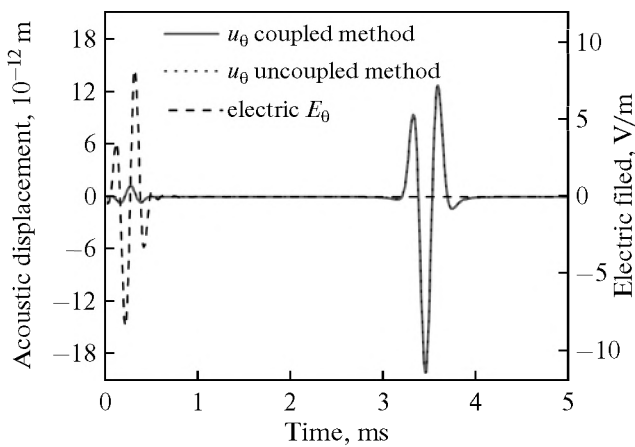
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Fig. 3. Transient full waveforms of electric field near the borehole wall and acoustic displacement of the borehole wall for the slow formation.
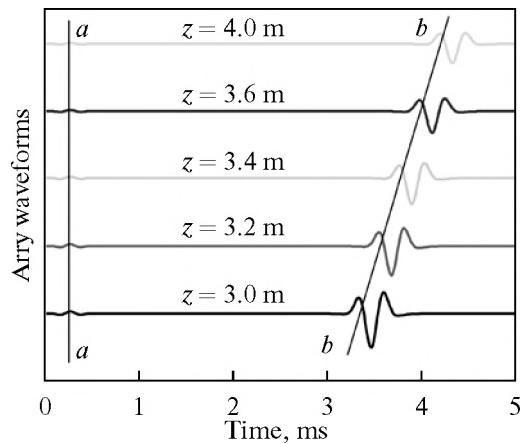

Fig. 4. Transient full waveforms of acoustic displacement of the borehole wall for the slow formation when the distance to the source z varies from 3.0 to 4.0 m.

ferent wave packets of particle displacement in Fig. 4 shown by lines a—a and b—b. Two wave packets are originated at the borehole wall through conversion of EM energy to acoustic energy. Once generated, they propagate independently of the electric field. The wave packets (line a—a) are the EM-accompanying acoustic wave groups. The difference in time between arrivals at the receivers shows that the second wave packets propagate with shear wave velocity about 935 m/s. Although the conversion efficiency is very low and such technique will be dependent on efficiency of detector and amplitude of source. It might be possible to obtain shear wave velocity especially in slow formation.

CONCLUSIONS

Because of uncoupling of SH waves with P-SV waves, a simple and forthright way to get shear waves information might be possible, especially for soft or slow formation whose shear wave velocity is lower

than the velocity of borehole fluid. The properties of TE-SH seismoelectric wave fields in a fluid filled borehole surround by a porous media are studied. We considered the electroseismic wave fields excited a vertical magnetic dipole in a borehole. Two methods are used to simulate, one is the coupled method based on Pride model and another is the uncoupled method. The full waveforms of electromagnetic wave induced SH waves excited by VMD source in the time domain propagation in borehole are simulated and analyzed. It might be possible to use SH-TE or TE-SH logging to measure shear wave velocity directly.

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